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The Two Sector Model of Learning-By Doing and Productivity Differences

Summary: This paper proposes that even when all countries have access to common technology frontier and can use the technologies which are fully appropriate to their needs, there will still be productivity differences across countries depending on their relative skill endowments. To illustrate this view, we have constructed a two sector model of productivity differences in which the level of technology is determined endogenously depending on the aggregate capital externalities. The relative supply of skilled and unskilled labor determines the direction of technical choices of the countries and differences in these relative factor supplies lead to cross-country income differences combined with the fact that capital is more productive in the advance of the skilled labor complement technologies than in the unskilled labor complement technologies.

Key words: Productivity differences, Technological change, Skilled/unskilled labor.

JEL: O33, O40, J24.

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Generally speaking, the models developed by the growth theorists mainly attempt to answer this fundamental question: What determines the large disparities in *per capita* income across countries? Despite the lack of a unifying theory, there exists large number of theories that investigate the determinants of economic growth. Initially, the literature emphasizes the importance of the physical capital and human capital on process of growth, but over the last two decades much of the focus has shifted to the role of technological advance as an engine of economic growth.

Many economists in the present believe that most cross-country differences in *per capita* output stem from differences in the level of total factor productivity, rather than differences in the levels of inputs (Peter J. Klenow and Andres Rodriguez-Claire 1997; Edward J. Prescott 1997; Robert E. Hall and Charles I. Jones 1999). It is widely argued that differences in technological knowledge in many cases are the main source of these output differences. Indeed, economic historians have long highlighted the importance of technological process as a main source of productivity and economic growth (Alexander Gerschenkron 1952; Moses Abramowitz 1986). In this context, a large number of endogenous growth theories which take their roots from Robert M. Solow (1956) influential paper bring up the prominence of the same phenomenon with more formal technics (Paul M. Romer 1986, 1990; Gene M. Grossman and Elhanan Helpman 1991; Philippe Aghion and Peter Howitt 1992). While complete agreement is not possible, one can conclude that there is a near consensus

in the literature on the view that cross-country differences in income *per capita* cannot be understood solely on the basis of differences in physical and human capital.

In this sense, it is important to understand why these total factor productivity disparities take place across countries. In this study, we propose a model in which there exists some degree of productivity differences across countries which differ in their relative skill endowments.

The rest of the paper is organized as follows. Section 1 briefly reviews the related literature. Section 2 presents the model and characterizes the equilibrium properties of our hypothetical economy. Section 3 discusses how the model generates cross country productivity differences. Finally, in Section 4 we present conclusions.

1. Literature Review

Why do total factor productivity disparities take place across countries? The theoretical literature, at this point offers two broad classes of explanations. A first explanation relies on the view that technology diffusion across borders may be far from perfect. Some developing countries may not be entirely benefited from new technologies because of the various barriers to technology transfer which keeps the older technologies in use. Many empirical studies report that there exists huge cross-country disparities in the degree of diffusion and adoption of technologies which in turn leads to large cross-country differences in income *per capita* (Diego Comin, Bart Hobijn, and Emilie Rovito 2006; Chang-Tai Hsieh and Klenow 2007). Barriers to technology transfer in most cases involve various costs affecting the diffusion and adoption of the new technology such as barriers to international trade (Rodriguez-Claire 1996; Yao Li 2010), limited human capital endowments (Richard R. Nelson and Edmund S. Phelps 1966; Gregory D. Wozniak 1987; Jörg Mayer 2001) and required R&D expenditures to install new technologies (Wesley M. Cohen and Daniel A. Levinthal 1989; Miroslav Verbic et al. 2011). More recent literature on the topic, on the other hand, particularly emphasize the effects of policy barriers on diffusion and adoption of new technologies. For example, Comin and Hobijn (2009) propose that lobbying activities which are conducted by old technology producers competing with new technology producers affect the speed of diffusion through its effect on the political cost of erecting barriers. Details of this literature can be found in Mancur Olson (1982), Joel Mokyr (1990), Stephen L. Parente and Prescott (1994, 1999, 2006), Benjamin Bridgman, Igor D. Livshits, and James C. MacGee (2004).

Regardless of what hampers process of technology transfer, if the new technology is sufficiently superior to the old one, all other things being equal, the productivity differences will exist between such countries which raise the barriers against the diffusion of new technologies and which do not.

The second explanation, on the other hand, (which is often called as appropriate technology argument), stresses that these productivity differences take place because the technologies developed in advanced countries may not be fully appropriate to the developing countries' needs (Ernst Friedrich Schumacher 1973; Frances Stewart 1977; Daron Acemoglu and Fabrizio Zilibotti 2001). According to advocates of this view, the characteristics of many technologies are designed to fit in with the economic and institutional structures of the economy which they were designed for.

In all sorts of respects, technologies developed in advanced countries reflect their conditions and require the more or less similar conditions for an efficient use where they are installed. According to Stewart (1977) for example these include the existence of relatively large scale of production, certain level of income, linked technics and/or of inputs of a particular quality in the rest of the economy. Schumacher (1973) who is in some sense the initiator of the appropriate technology argument also argues that technologies which are particularly developed for mass production and are transferred by large, capital intensive projects are inappropriate to the needs of the poor countries due to weak institutional structure, inappropriate inputs, unskilled workers and the shortage of management skills.

More recently, this argument has been reconsidered in two influential papers by Susanto Basu and David N. Weil (1998) and Acemoglu and Zilibotti (2001). Basu and Weil construct a model in which technologies are specific to particular capital/labor ratio. Consequently, a country may not efficiently be able to use a new technology until it reaches a level of development which is required for efficient use of this technology. In a similar motivation, Acemoglu and Zilibotti (2001) proposes that technology developed in the North creates technology-skill mismatch in the South because differences in the supply of skills in these two groups of countries. Thus, when the technologies developed in more advanced countries are transferred to developing countries, they may imply a relatively small productivity gain and even may leads to the distortions. In other words, free access to the most advanced technologies in the world does not guarantee to enable developing countries to catch-up to the advanced countries. The appropriate technology view has also received some empirical support in several studies. For example, Bart Los and Marcel P. Timmel (2005) test the validity of the arguments of Basu and Weil (1998) and find that absorption of technologies new to a country is a costly and slow process. Similarly, Michal Jerzmanowski (2007) find the evidence that countries with an inadequate blend of inputs ie., the country's relative stocks of physical and human capital, are unable to pick all the benefits of the most advanced technologies although there are no any barriers to technology transfer.

The models developed so far, in some fundamental sense, fall into one of these two broad categories described above. In this study, we propose an additional consideration for the topic. More explicitly, we argue that even when all countries have access to common technology frontier and can use the technologies which are fully appropriate to their needs, there will still be productivity differences across countries depending on their relative factor endowments.

To illustrate this view, we consider an economy consisting of two goods (two sectors) one of which is unskilled labor intensive, while the other uses the skilled labor intensively. The unique final good is produced by combining the output of these sectors and technologies are specific to each goods which are determined by the aggregate capital stock of the economy as in Romer (1986). At this point, we make a plausible assumption that one unit of capital leads to much more increase in the technology used in the skilled labor intensive sector than unskilled labor intensive sector. Then it is easy to predict that as long as the elasticity of substitution between skilled and unskilled labor is large, in a country that is skilled labor abundant, the

fraction of capital employed in the skilled-labor intensive sector will be large in equilibrium and because of the assumption we have made, *per capita* aggregate output will be higher in this country than those with unskilled labor intensive.

It is noteworthy that in our model, there is no inappropriateness on the choice of technology of the countries. Indeed, the model is based on the idea that the technology choice is solely determined by the relative factor supply of the countries and that in turn this choice can lead to productivity differences across countries under certain circumstances. In this context, the technologies are produced appropriately by the countries in the sense that they reflect the relative skill endowments of these countries. For this manner our paper is closely related to Acemoglu (2002) who proposed the term “directed technical change” to illustrate this kind of technological choice. However, different from the models of Basu and Weil (1998), and Acemoglu and Zilibotti (2001), in our model, there are no technology-skill or technology-capital/labor mismatches across countries. Also, all countries have access to the same set of technologies, that is, the technologies diffuse without any bound across borders. It should be noted that our argument can be viewed as a complementary explanation to the other arguments described above. If there are huge barriers to technology transfer across countries, argument of barriers to technology diffusion will be more relevant. Conversely, if the countries have access to new technologies without any bound, (and *per capita* income differences still take place) our argument and appropriate technology view become more relevant.

2. The Model

2.1 The Production Structure

The economy consists of two sectors (two goods) and the unique final good is produced by combining the outputs of these sectors. The production technology of the final good takes the CES form with elasticity of substitution $\varepsilon \in [0, \infty)$:

$$Y = [\lambda (Y_L)^\sigma + (1-\lambda) (Y_H)^\sigma]^{1/\sigma} \quad (1)$$

where Y_L and Y_H denote the unskilled labor and skilled labor intensive goods respectively. $\lambda \in (0,1)$ is the distribution parameter which captures the relative importance of these two goods in the production of the final good.

CES specification of the production function allows us to analyze the case of imperfect substitution between factors and goods. When $\sigma = 1$ the two goods are perfect substitutes and when $\sigma = 0$ the elasticity of substitution between the two goods is equal to 1. In this context, two goods are imperfect substitutes as long as $\sigma < 1$. Accordingly, when $0 < \sigma < 1$, we refer to the goods as good substitutes and when $\sigma < 0$, we refer to the goods as good complements.

In this context, the value of the elasticity of substitution between unskilled and skilled workers is of particular importance for our results. At the moment, most of the empirical studies report strong evidence that the elasticity of substitution between these two types of workers is likely in the range between 1 and 2. George E.

Johnson (1970), for instance, estimates the elasticity of substitution between more and less educated workers as 1.34, using a cross section of U.S. states in the year of 1960. As one of the most cited paper in the field, Lawrence F. Katz and Kevin M. Murphy (1992) estimate the elasticity of substitution between college and high school labor as about 1.41 using data of U.S. for the period 1963-97. Using the similar data for the period 1960-90, James J. Heckman, Lance Lochner, and Christopher Taber (1998) find that the value of the related elasticity is 1.44 which is remarkably close to estimation documented by Katz and Murphy (1992). More recently, Antonio Ciccone and Giovanni Peri (2005), estimate the long-run elasticity of substitution between more and less educated workers at the U.S. state level using the data between the years 1950 and 1990. They also conclude that the estimated elasticity is 1.5. Kevin M. Murphy, Craig Riddell, and Romer (1998) obtain an estimate of 1.36 using the time-series data of Canada and Peter R. Fallon and Richard G. Layard (1975) obtain the value of 1.31 for the cross-section of countries. Johnson (1997) discusses the value of the elasticity of substitution between the these two types of labor, and concludes that this value is in the neighborhood of 1.5. Similarly, David Autor, Katz, and Alan B. Krueger (1998) argue that the elasticity of substitution is likely in the range between 1 and 2.

Hence, based on the existing empirical evidence, we will usually present the analysis for the both case, but will focus on and derive our main conclusion for the case of $0 < \sigma < 1$ which is the empirically relevant case.

We assume that two goods are produced competitively with the Cobb-Douglas technology using labor and capital:

$$Y_{iL} = (A_L L_i)^{1-\alpha} K_{iL}^\alpha \quad (2)$$

and

$$Y_{iH} = (A_H H_i)^{1-\alpha} K_{iH}^\alpha \quad (3)$$

where L and H are the total supplies of unskilled and skilled labor and A_L and A_H denote the level of technology (which is the same across firms) in the production of unskilled labor and skilled labor intensive goods respectively. Similarly, K_L and K_H denote the amount of capital employed in these two sectors.

Notice that the production functions (2) and (3) exhibit constant returns to scale in labor (L, H) and capital (K_L, K_H) for given “ A ” and increasing returns to scale in capital, labor and the technology index; A . Since the production functions of these two goods exhibit constant returns for given A which producers take as given, producers of these goods are competitive. Notice also that, the share of capital in output is the same for both sectors. Thus, these sectors only differ in the use of unskilled labor or skilled labor in their productions.

2.2 Learning-by Doing and Technology

Technological progress takes the form of expansions in A . We formulate the technology index and its evolution as in Romer (1986) in which the knowledge stock of the economy is assumed to be proportional to the capital stock of the economy.

More specifically, the state of the technology at any date is given by:

$$A_L = \beta_L \cdot \kappa_L \quad (4)$$

and

$$A_H = \beta_H \cdot \kappa_H \quad (5)$$

where κ_L and κ_H are the aggregate stock of capital used in unskilled and skilled labor intensive sectors respectively. β_L and β_H denote the productivities of capital in the evolution of technology in the relevant sectors.¹

This specification of technological advance depends mainly on the Kenneth J. Arrow (1962) well-known concept of learning-by doing.² As the capital stock increases, workers obtain new skills and become more productive as a result. In other words, the production process itself can lead to the advance of the technology.

What makes this formulation also useful is that although the level of technology is given for the individual firm since they cannot affect the aggregate stock of capital, this stock of technology is accumulated endogenously in the economy as a whole. Hence, the production function exhibits constant returns to scale at the firm level, but at the aggregate level it exhibits increasing returns to scale which ensures endogenous growth of the economy.

Finally, we make a key assumption that aggregate capital is more productive in the advance of the skilled labor complement technology than in the unskilled labor complement technology. Put another way, we assume that

$$\beta_H > \beta_L.$$

The intuition behind this assumption is quite straightforward: Imagine that assembly-line production process which uses relatively unskilled workers on the one hand and information technology based complex production process which uses chiefly skilled workers on the other. As one might think, there will be possibly more things for workers to experience in the latter than the former. Similarly, Basu and Weil (1998) propose that the *maximum* level of technology increases with capital, which captures the idea that technologies have increasingly high potential at higher levels of development.

¹ Note that these equations assume that there is constant return to capital in the advance of technology. This is assumed chiefly for simplicity without loss of any generality. Instead, we could assume that $A = \beta \kappa^\theta$, $0 < \theta < 1$ which implies diminishing return to capital. In this case, even though the static analysis (which is our main interest) would not change, dynamic properties of the system leads to semi-endogenous growth.

² Romer (1986) uses this specification mainly to introduce to the non-rivalry nature of the knowledge into the production function. See also, Robert Wilson (1975), Douglas A. Irwin and Pete Klenow (1994), Boyan Jovanovic and Yawb Nyarko (1994).

2.3 Total Output of the Two Sectors

Let us write total output of each sector in more explicit form. To do this, substituting the technology equations (4) and (5) into (2) and (3), we obtain the total supplies of unskilled and skilled labor goods as

$$Y_{iL} = (\beta_L \kappa_L L_i)^{1-\alpha} K_{iL}^\alpha \quad (6)$$

and

$$Y_{iH} = (\beta_H \kappa_H H_i)^{1-\alpha} K_{iH}^\alpha \quad (7)$$

Notice that these production functions exhibit constant returns to scale in K_i and L_i for given κ and therefore all goods markets are competitive. However, at the aggregate level increasing returns to scale take place because of productive externalities of aggregate capital stock of the economy.

Then, the aggregate production function is straightforward to derive. In equilibrium, we must have $K_i = K$ for all i , since all firms are assumed to be identical. Thus, dropping the index i , the aggregate production function can be written as

$$Y_L = (\beta_L \kappa_L L)^{1-\alpha} K_L^\alpha \quad (8)$$

and

$$Y_H = (\beta_H \kappa_H H)^{1-\alpha} K_H^\alpha \quad (9)$$

Notice that capital stock, K enters the production functions in two ways: First, it enters the production function as an input; (which is captured by K). Second, it affects productivity with its externalities (which is captured by κ).

Now substituting $\kappa_L = K_L$ and $\kappa_H = K_H$ into production functions above, we obtain:

$$Y_L = (\beta_L L)^{1-\alpha} K_L^\alpha \quad (10)$$

and

$$Y_H = (\beta_H H)^{1-\alpha} K_H^\alpha \quad (11)$$

With these equations, it is more convenient to see that these production functions do not exhibit diminishing returns to capital because of the existence of capital externalities we described above.

Finally, combining (10) and (11) with (1), the aggregate output of the economy can be written as

$$Y = [\lambda ((\beta_L L)^{1-\alpha} K_L^\alpha)^\sigma + (1-\lambda)((\beta_H H)^{1-\alpha} K_H^\alpha)^\sigma]^{1/\sigma} \quad (12)$$

Note also that the elasticity of substitution between the factors L and H can be calculated from this function as:³

$$\psi = 1/(1 + \sigma\alpha - \sigma)$$

where ψ is the elasticity of substitution between skilled and unskilled labors.

This definition implies that $\psi > 1$ if and only if $\sigma > 0$ (and $\psi < 1$ if and only if $\sigma < 0$). In other words, skilled and unskilled labor is good substitutes only if the skilled and unskilled labor intensive goods are good substitutes. The similar pattern is also true for the case of complementarities between the factors.

2.4 Static Equilibrium

Obviously, the equilibrium in this model consists of labor and capital allocation and factor and good prices in which firms maximize profits and markets clear. Moreover, it includes consumption and savings decisions which maximize consumer utility. Here, instead of investigating a complete characterization of the equilibrium, we will focus on the particular properties of equilibrium which our model is concerned.

First, since Y_L and Y_H are produced competitively, their relative prices are equal to the relative value of their marginal product⁴:

$$\frac{p_L}{p_H} = \frac{\lambda}{(1-\lambda)} \left(\frac{Y_L}{Y_H}\right)^{\sigma-1} \quad (13)$$

where p_L and p_H are the prices of the Y_L and Y_H respectively.

This equation states a common result that as the supply of Y_L relative to Y_H increases, its relative prices decreases.⁵

Since product markets are competitive, firms in the unskilled-labor intensive sector face the following maximization problem. (The maximization problem facing firms in the skilled-labor intensive sector is similar.)

$$\max p_L Y_L - w_L L - r_L K_L$$

where w_L is wage rate of unskilled labor and r_L is the rental price of the capital employed in unskilled-labor intensive sector.

The first-order conditions of these problems state that since factor markets are assumed to be competitive, their prices are equal to the value of their marginal products. Thus, dividing the first-order condition for maximization problem of L-intensive sector by the condition for maximization in H-intensive sector, we get

³ The derivation of this elasticity is given in the Appendix.

⁴ See the Appendix for a more explicit presentation.

⁵ Note that it is always true that $(\sigma - 1) < 0$, since $\sigma \leq 1$.

$$\frac{r_L}{r_H} = \frac{\beta_L p_L L^{1-\alpha}}{\beta_H p_H H^{1-\alpha}} \quad (14)$$

This equation simply states that relative rental price of the capital, r_L/r_H depends positively on the relative good prices, p_L/p_H and on the relative factor endowments, L/H . As might be expected, if the price of one good increases relative to the other, the demand for capital employed in the production of this good also increases. This, in turn, increases the rate of return to this capital.

Then, substituting (13) for p_L/p_H and using some algebra, we can express r_L/r_H as a function of the relative factor supply L/H and the relative output Y_L/Y_H .⁶

$$\frac{r_L}{r_H} = \left(\frac{\lambda \beta_L}{(1-\lambda) \beta_H} \right) \left(\frac{L}{H} \right)^{1-\alpha} \left(\frac{Y_L}{Y_H} \right)^{\sigma-1} \quad (15)$$

Finally, setting $r_L = r_H$ since competition equates the prices of capital which is assumed to be identical for two goods, and substituting (10) and (11) into (15), we obtain the share of capital employed in the unskilled labor intensive sector as

$$S_{KL} = \frac{K_L}{K} = \left[1 + \left(\frac{\beta_L}{\beta_H} \right)^{(\sigma/\sigma-1)} \left(\frac{L}{H} \right)^{(1-\alpha)(\sigma/\sigma-1)} \left(\frac{\lambda}{1-\lambda} \right)^{1/\sigma-1} J \right]^{-1} \quad (16)$$

Obviously, $S_{KH} = 1 - S_{KL} = \frac{K_H}{K}$.

Equation (16) shows the capital share in the unskilled labor intensive sector as a function of parameters β_j , λ and the relative supply of labors, L/H . The elasticity of substitution between skilled and unskilled labor determine how these variables affect this capital share.

Let us first start the case of good substitubility between factors which is also the empirically relevant case. When $0 < \sigma < 1$, i.e. the two goods (two types of labors) are good substitutes, the fraction of capital employed in the unskilled-labor intensive sector is *increasing* in the β_L/β_H and L/H . The intuition for this result is straightforward: When two goods are good substitutes, an increase in the relative supply of factors and the technology parameter attached to them in favor of one sector, would increase demand for the other input in this sector in order to increase its output.

⁶ There is also a first-order condition with respect to labor:

$$\frac{w_L}{w_H} = \frac{\lambda}{(1-\lambda)} \left(\frac{\beta_L}{\beta_K} \right)^{(1-\alpha)\sigma} \left(\frac{L}{H} \right)^{(1-\sigma)[(\alpha\sigma/\sigma-1)-1]} \left(\frac{K_L}{K_H} \right)^\sigma$$

but it plays no role in our model.

Conversely, when $\sigma < 0$, i.e. the two goods (two types of labors) are good complements, the fraction of capital employed in the unskilled-labor intensive sector is *decreasing* in the β_L / β_H and L/H .

Finally, note that S_{KL} always positively depends on the distribution parameter $\lambda / (1 - \lambda)$ irrespective of the elasticity of substitution between the two goods.

2.5 Comparative Static Analysis

To make more progress, it is useful to determine how these capital shares change with capital accumulation, and with any change in the ratio of different types of labors, L/H .

Accordingly, differentiating the capital share, S_{KL} with respect to K and L/H , we get:

$$\frac{\partial S_{KL}}{\partial K} = \frac{\partial S_{KH}}{\partial K} = 0 \tag{17}$$

$$\frac{\partial S_{KL}}{\partial(L/H)} = \frac{-[(1-\alpha)(\sigma/\sigma-1)(\frac{\beta_K}{\beta_L})^{\sigma/\sigma-1}(\frac{L}{H})^{(1-\alpha)(\sigma/\sigma-1)-1}(\frac{\lambda}{1-\lambda})^{1/\sigma-1}]}{[1+(\frac{\beta_K}{\beta_H})^{\sigma/\sigma-1}(\frac{L}{H})^{(1-\alpha)(\sigma/\sigma-1)}(\frac{\lambda}{1-\lambda})^{1/\sigma-1}]^2} \tag{18}$$

Equation (17) states that the fraction of capital employed in these two sectors does not change with the capital accumulation irrespective of the value of σ .

Equation (18) is a more clear representation of the patterns discussed above. Let us summarize our main results with a more convenient way:

$$S_{KL} = \Phi \left\{ \left(\frac{L}{H}\right), \left(\frac{\beta_L}{\beta_H}\right), \left(\frac{\lambda}{1-\lambda}\right) \right\}, \quad \text{when } 0 < \sigma < 1$$

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$$S_{KL} = \Phi \left\{ \left(\frac{L}{H}\right), \left(\frac{\beta_L}{\beta_H}\right), \left(\frac{\lambda}{1-\lambda}\right) \right\}, \quad \text{when } \sigma < 0.$$

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3. Productivity Differences

The model presented so far constructs a relationship between the fraction of capital employed in each sector and the relative skill endowments. We are now in a position to apply this analysis to understand how productivity differences may take place even when countries develop technologies that are appropriate to their needs. As one might expect, this model is well-suited to generate cross-country productivity differences under some plausible assumptions. To illustrate this, let us suppose there are

two countries which only differ in their relative skill endowments. More specifically, we assume that,

$$\frac{H^a}{L^a} > \frac{H^d}{L^d}$$

that is the advanced country is relatively skilled labor abundant while the developing country is relatively unskilled labor abundant. In other words, we consider two economies with identical initial conditions, but with different skill endowments.

Additionally, we have assumed that aggregate capital is more productive in the advance of the skilled labor complement technology than in the unskilled labor complement technology. Thus,

$$\beta_H > \beta_L$$

Now, using the equation (4.14), consider the *per capita* output which is given by

$$y = \frac{Y}{L+H} = \frac{[\lambda((\beta_L L)^{1-\alpha} K_L)^\sigma + (1-\lambda)((\beta_H H)^{1-\alpha} K_H)^\sigma]^{1/\sigma}}{L+H} \quad (19)$$

For simplicity, suppose that these two countries have also the same size of population, that is, $(L+H)^a = (L+H)^d$. Then, equation (16) implies that the fraction of capital employed in the skilled-labor intensive sector will be much larger in the advanced country than in the developing country. This, combined with the assumption of $\beta_H > \beta_L$, implies that the level of technology in the advanced country will be higher than in the developing country. Consequently, as might be seen in the equation (19), output *per capita* in the advanced country will be higher than in the developing country.

It should be noted that the current model, as the other models featuring capital externalities, suggest that higher saving rate in physical capital not only increases capital per capita, but also technological advance. However, we propose that there is heterogeneity in learning-by doing effect across sectors of the economy. More explicitly, we assume that aggregate capital is more beneficial in the advance of the skilled labor complement technology than in the unskilled labor complement technology. Then, we show that the fraction of capital employed in both sectors are determined by the relative skill supply of the economy.

For completeness, we characterize the long-run growth rate of this economy. Assume that the asymptotic growth rates g_H and g_L exist.

Then, differentiating the production function for the final good (1) with respect to time we obtain the growth rate of output as:

$$g = \frac{[\lambda Y_L^\sigma g_h + (1-\lambda) Y_H^\sigma g_l]}{[\lambda Y_L^\sigma + (1-\lambda) Y_H^\sigma]} \quad (20)$$

This equation implies that if $0 < \sigma < 1$, then as $t \rightarrow \infty$, $g = \max\{g_l, g_h\}$.⁷

As a result, when two goods (and factors) are good substitutes, the sector that is growing more faster will determine the asymptotic growth rate of aggregate output, and the converse applies when two goods are good complements. Recall that for the empirically relevant case of the elasticity of substitution less than one, this result states that a country with unskilled-labor abundant should rise the growth rate of skill levels of their labor force to catch-up the countries with skilled-labor abundant.

4. Conclusion

Many economists in the present argue that differences in technological knowledge in many cases are the main source of cross-country income differences. Motivated by this consideration, this paper demonstrates that the relative supply of skilled and unskilled labor determines the technical choices of the countries and differences in these relative factor supplies in turn lead to cross-country income differences. The intuition for this argument relies on the view that learning-by doing effect in technological advance has more potential in skilled labor intensive production processes.

An important aspect of our analysis is that it proposes a third explanation of cross-country income differences which complements the existing theories-barriers to technology adoption and appropriate technology-in one respect. Nevertheless, it is also noteworthy that each argument has a relative importance depending on the observed specific case. For instance, as one may expect, developing countries often face significant barriers to technology adoption. Thus, if these barriers are huge, we can conclude that the argument of barriers to technology diffusion will naturally be more relevant. Conversely, if the countries have access to new technologies without much bound, (and *per capita* income differences still take place) our argument and appropriate technology view become more relevant.

There nevertheless remains a need for empirical validation of the premises of the model that a conclusion is drawn from. Such an empirical analysis should make evident at least two points. First, there is need to confirm the argument that the learning-by doing effect is much stronger in the skilled labor intensive sectors than in the other sectors of the entire economy. Second, if this is the case, then we should quantitatively examine the importance of this fact in the explanation of cross-country income differences. This type of empirical strategy, within countries, inherently requires the careful classification and examination of the sectors based on their relative factor endowments. These issues provide the fundamental directions for future research.

⁷ Similarly, when $\sigma < 0$, then $t \rightarrow \infty$, $g = \min\{g_l, g_h\}$.

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Appendix

Derivation of the Elasticity of Substitution Between Skilled and Unskilled Labor

Here we present how to derive the elasticity of substitution between skilled and unskilled labor from the production function.

Let us consider the following production function

$$f(x_1, x_2) = y \quad (1)$$

The technical rate of substitution (*TRS*) between x_1 and x_2 is calculated as

$$TRS = - \frac{\partial f / \partial x_1}{\partial f / \partial x_2} \quad (2)$$

which determine how one has to adjust x_2 to keep output constant when x_1 changes by a small amount.

Then, elasticity of substitution between x_1 and x is given by the following formula:

$$\psi = \frac{d \ln(x_2 / x_1)}{d \ln TRS} \quad (3)$$

Recall that the production function in the model is given by

$$Y = [\lambda ((\beta_L L)^{1-\alpha} K_L)^\sigma + (1-\lambda) ((\beta_H H)^{1-\alpha} K_H)^\sigma]^{1/\sigma} \quad (4)$$

Using the formula (2), *TRS* between L and H can be written as:

$$TRS = - \left(\frac{\lambda}{1-\lambda} \right)^\sigma \left(\frac{\beta_L}{\beta_H} \right)^{(1-\alpha)\sigma} \left(\frac{L}{H} \right)^{\sigma-\sigma\alpha-1} \left(\frac{K_L}{K_H} \right)^\sigma \quad (5)$$

Finally, using the formula (3), we find the elasticity of substitution between L and H as:⁸

$$\psi = 1 / (1 + \sigma\alpha - \sigma) \quad (6)$$

which implies that $\psi > 1$ if and only if $\sigma > 0$.

⁸ First, solve equation (5) for L/H and take logarithm of the both side ($\ln(L/H)$). Then differentiating $\ln(L/H)$ with respect to $\ln TRS$ gives the desired result.