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On the Concept of Market Concentration, the Minimum Herfindahl-Hirschman Index, and Its Practical Application

Summary: The paper analyses the phenomenon of market concentration in the context of the most popular industrial organization approaches: the structure-conduct-performance (SCP) approach and the efficiency hypothesis (EH). The theory of market concentration evaluation is briefly presented, followed by an analysis of the main concentration indicators and their application in recent empirical research. The main problem in the practical application of the Herfindahl-Hirschman index (HHI) – the necessity to incorporate the market shares of all the enterprises in the market – is further addressed and the issue of calculating the minimum value of the HHI is analysed from the theoretical point of view and by modelling hypothetical markets.

Key words: Market structure, Market concentration, Herfindahl-Hirschman index (HHI).

JEL: D21, D22, D40, D42, D43, D47.

Scholars of economics (e.g. Israel M. Kirzner 1963; Milton M. Shapiro 1985; Ludwig von Mises 1996, 1998) associate highly concentrated markets with a potential threat of cartels or other agreements on the coordination of actions and consumer-inefficient equilibrium even in the case of fair competition. The issue of quantitative evaluation of market concentration is important both for scientists implementing empirical research on the relationship between market structure and enterprise performance and for government authorities for the purpose of antitrust market regulation.

The modern scientific literature on the subject offers dozens of indicators measuring market concentration; however, none of them is considered to be perfect (Romualdas Ginevičius and Stasys Čirba 2007, 2009). The concentration ratio (*CR*) and Herfindahl-Hirschman index (*HHI*) have become the most popular indicators for measuring market concentration in empirical research due to the simplicity of their calculation. However, the latter indicator requires the market shares (or sales volumes) of all the companies in the market to be known, which is its main difficulty in practical application.

The *problem* of this paper is the application of the Herfindahl-Hirschman index in empirical research incorporating the data of the largest companies in the market or industry only. The *aim* of the research is to analyse theoretically and practically the issue of calculating the minimum value of the Herfindahl-Hirschman index based on the market shares of the largest companies, considering and critically assessing the advantages and disadvantages of such approximation.

1. Measuring Market Concentration: Theory and Practice

It is not common in recent empirical research for the market concentration to become the ultimate indicator to which research is devoted (e.g. Jonas Čepinskis and Kristina Gancevskaitė 2008; Ginevičius and Algirdas Krivka 2009; Silvia Marginean and Ramona Toma 2011); rather, in the framework of industrial organization, concentration is considered to be the quantitative indicator characterizing the market structure in research following the structure-conduct-performance (*SCP*) approach (Joe S. Bain 1956, 1959) or the efficiency hypothesis (*EH*) (Harold Demsetz 1973; Sam Peltzman 1977) approach.

Under the *SCP* (in its original form), market concentration is an exogenous variable of market structure, which indirectly, through enterprise conduct (possible collusive behaviour), affects the industry performance measured by the level of output, profitability, or other kinds of indicators (Vilma Deltuvaitė and Vilda Gižienė 2007; Čepinskis and Gancevskaitė 2008; Valdonė Darškuvienė and Jonė Šakalytė 2009; Marginean and Toma 2011; Thao N. Nguyen and Chris Stewart 2013). Such an approach provides theoretical grounds for antitrust regulation aimed at controlling mergers and acquisitions to limit the market shares of the largest or dominant companies.

Alternatively, the efficiency hypothesis approach suggests that a positive relationship between concentration and profitability does not necessarily reflect collusive behaviour by several firms; rather, it shows that larger firms earn higher profits by performing efficiently. In such a case, governmental intervention is inappropriate, since it might impose penalties on efficient firms and discourage the proper functioning of the market mechanism (Čepinskis and Gancevskaitė 2008; Nguyen and Stewart 2013). In EH-based models market concentration is considered to be an endogenous variable – because the market share of the company depends on its performance – as is the concentration of the whole market (Deltuvaitė and Gižienė 2007).

The concentration measurement is based on the concept of the concentration curve (Ginevičius and Čirba 2007, 2009), drawn in a two-dimensional system of coordinates, in which the abscissa carries the number of market players in descending order of their magnitudes while the ordinate is marked with the cumulative weighted players' market shares. The concentration indicators, based on this concept, are discrete or cumulative: the former evaluate only the number of concentration curve points, while the latter take account of them all (Ginevičius and Čirba 2007; Ginevičius, Vladislavas Petraškevičius, and Jolita Šimkūnaitė 2010). Table 1 provides a brief overview of the empirical application of concentration indicators in a number of recent studies.

Table 1 An Overview of the Application of Concentration Indicators in Recent Empirical Research

Sources	**Concentration indicators applied**	**Brief descriptions of the research**
Samuel Y. Akomea and Michael Adusei (2013)	CR_3, CR_4, CR_5, CR_6, *HHI*	The paper investigates the concentration levels of the banking industry in Ghana in 2003-2010.
Vivien Beattie, Alan Goodacre, and Stella Fearnley (2003)	CR_4, CR_8	The paper explores the concentration of the UK and the US audit markets and its impact on competition.
Jason Beck, Frank Scott, and Aaron Yelowitz (2012)	CR_4, *HHI*	A study of the concentration and market structure of the US local real estate market.
Alfredo M. Bobillo, Miguel A. F. Temprano, and Fernando T. Gaite (2004)	An integrated indicator based on the *HHI*, Hannah-Kay, and Theil Entropy index	The study develops a systematic analysis of the concentration and inequality levels of 20 Spanish industries over the period of 1990-2001.
Čepinskis and Gancevskaitė (2008)	CR_3, CR_5, *HHI*, *H*-statistics (non-structural approach)	A study on the concentration and competitiveness of the Lithuanian life insurance market in the period of 2001-2007.
Darškuvienė and Šakalytė (2009)	CR_3, $CR_{10\%}$, *HHI*	The paper explores the changes in the securities market concentration in Europe based on the analysis of the formation of networks among European stock exchanges.
Deltuvaitė (2009)	CR_3	The paper explores the impact of banking sector concentration on its stability (Z-index characterizing the probability of bankruptcy) and performance (characterized by a number of widely applied profitability indicators) in 162 countries around the globe in 1987-2007.
Deltuvaitė and Gižienė (2007)	CR_1, CR_3, CR_5, *HHI*, *H*-statistics	A study of the relationship between competition, concentration, and efficiency in the Lithuanian banking sector in 2000-2005.
Ginevičius, Petraškevičius, and Šimkūnaitė (2010)	*HHI*, *GIN*, *GRS*	The paper investigates the influence of market concentration on enterprises' financial state and performance indicators in Lithuanian industries in 2006.
Fred H. Hays, Stephen A. DeLurgio, and Arthur H. Gilbert Jr. (2009)	*HHI*	The study examines the impact of market concentration on the yield on earning assets and the cost of funds for commercial banks at the state level for 2006 and 2007.
Indrit Hoxha (2013)	CR_5, *HHI*	The study explores the effect of banking concentration and banking competition on the volatility of the growth of value added of the manufacturing sectors in developing countries.
Li Liu and Rosanne Altshuler (2013)	CR_4	The paper assesses the burden of the corporate income tax on wages under imperfect competition in the US industries.
Charles Z. Liu et al. (2012)	*HHI*	The study analyses the effect of digital conversion technology on the competitiveness of the flash memory card market and among other things checks the hypothesis that the market concentration decreases as the adoption of digital converters increases.
Marginean and Toma (2011)	CR_5, CR_{20}	The authors assess the level of concentration of Romanian industries in the period of 1996-2004, focusing on the factors that determine the differences in concentration between industries.
Judith Mariscal (2009)	*HHI*	The paper investigates the process of consolidation in the mobile market of Latin America and, analyses the relationship between concentration, spectrum allocation, and prices.
Sami Mensi (2010)	*H*-statistics	The paper explores the use of the Panzar-Rosse statistic as a basis for empirical assessment of competitive conditions among Tunisian deposit banks.
Luigi Moretti (2012)	CR_3, CR_5, *HHI*	The study explores the relationship between the firm turnover in the economy and the banking sector concentration in the EU countries, considering the impact of bank financing as the key factor of new firm entry.
Jayanta N. Mukhopadhyaya, Malabika Roy, and Ajitava Raychaudhuri (2012)	*HHI*	The authors analyse the impact of deregulation and economic liberalization on the regional pattern of growth in the cement industry of India in terms of regional inequality as well as concentration.
Nguyen and Stewart (2013)	CR_3, CR_5, *HHI*	This study applies the *SCP* paradigm and the *EH* approach to investigate the impact of concentration and market shares on the performance of the Vietnamese banking system in 1999-2009.

Nina Ponikvar and Maks Tajnikar (2011)	*HHI*	The paper investigates the factors that affect the pricing policy in Slovenian manufacturing firms in terms of the mark-up size.
Vedapuri S. Raghavan (2013)	*HHI*	The paper investigates the consolidation in the US airline industry due to the Delta-Northwest merger in 2008.
Francesco D. Sandulli et al. (2012)	CR_4	The study explores the unclear relationship between industry structure (described by concentration, industry life cycle, and technology intensity) and open innovation.

Source: See column "Sources".

According to Table 1, the concentration ratio (*CR*) and Herfindahl-Hirschman index (*HHI*) are the most widely applied concentration indicators. One might argue that a more extended study is required for such kinds of conclusions, but it would hardly change the general tendencies of the latter indicators' popularity.

The concentration ratio, indicating the total (cumulative) market share of the *M* largest enterprises in the market, is calculated using the formula (Ginevičius and Čirba 2007; Ginevičius and Krivka 2009; Ginevičius, Petraškevičius, and Šimkūnaitė 2010):

$$CR_M = \sum_{i=1}^{M} d_i , \tag{1}$$

where d_i – the market share of the *i*-th enterprise; *M* – the number of enterprises analysed.

Usually the value of parameter *M* is less than the total number of enterprises in the market ($M \le N$); thus, the concentration ratio is considered to be a discrete concentration indicator (Ginevičius, Petraškevičius, and Šimkūnaitė 2010). However, from the point of view of the weighting scheme, it could be assumed that the attribute carriers from 1 to *M* are weighted with 1 while those starting from $M+1$ obtain the weights of 0 (Deltuvaitė and Gižienė 2007). There are no strict limitations on the range of values of parameter *M* (e.g. CR_5 is applied in the UK; CR_4, CR_8, and CR_{20} are used in the US), and the market shares of the largest companies only are required. Thus, the concentration ratio is simple to calculate and interpret; however, it does not show the distribution of the market shares within the range of $i \in [1,...,M]$ or the market shares of the players from $M+1$ to *N* (Ginevičius and Krivka 2009).

The Herfindahl-Hirschman index (*HHI*) is calculated using the formula (Ginevičius and Čirba 2007, 2009; Ginevičius and Krivka 2009; Ginevičius, Petraškevičius, and Šimkūnaitė 2010):

$$HHI = \sum_{i=1}^{N} d_i^2 , \tag{2}$$

where d_i – the market share of the *i*-th enterprise; *N* – the total number of enterprises in the market.

The values of the *HHI* are in the range from 0 (perfect competition in which the market shares of all the enterprises are close to 0) and 1 (pure monopoly). According to the antitrust regulation by the US Department of Justice, a market is considered to be competitive when $HHI \le 0.100$, moderately concentrated when

$0.100 < HHI \leq 0.180$, and highly concentrated in the case of $HHI > 0.180$ (Deltuvaitė and Gižienė 2007; Hays, DeLurgio, and Gilbert Jr. 2009). The *HHI* is a cumulative concentration indicator using the market shares of the enterprises as their weights, thus being the sum of the *squared* attribute values. This implies that larger enterprises gain an even bigger proportion of the value of the *HHI*, which has been criticized from the concentration curve perspective (Ginevičius and Čirba 2007, 2009); however, the *HHI* results in a higher value in the case of unequal market shares' distribution, which indirectly reflects the existence of market leaders and dominant players and thus a less competitive environment. The practical application of the *HHI* is limited because of insufficient availability of statistical data (the market shares of all the enterprises acting in the market are required), which is a common problem in empirical research.

Other concentration indicators, including entropy (Ann Horowitz and Ira Horowitz 1968), the exponential index (Peter K. Hani 1987), the Rosenbluth index (Gideon Rosenbluth 1961), and the Gini index (Corrado Gini 1936), are applied on an irregular basis only. However, the theoretical evidence from Ginevičius and Čirba (2007, 2009) indicates the drawbacks of all the well-known concentration measures; for example, all of them (perhaps with the exception of the Rosenbluth index) either ignore or pay little attention to the number of attribute carriers, which is one of the main free-market characteristics. The authors suggest their own *GIS* and *GRS* indicators, which have the smallest difference between the actual attribute carriers' weights and those used in the formula of the indicator.

Summarizing the brief study of the concentration measurement theory and practice, it has been determined that the concentration ratio and Herfindahl-Hirschman index are the most popular concentration indicators in empirical research, despite having their own drawbacks and limitations. The main problem of the empirical application of the *HHI* is the necessity to incorporate the market shares of all the enterprises in the market, which is even more complicated when analysing a number of markets or industries, the dynamics of concentration during a long period of time, or industries containing thousands of enterprises. As a possible solution to this problem, the next sections of the paper analyse the issue of calculating the minimum value of the *HHI* in empirical research, supported with calculations and a modelling technique.

2. Minimum Herfindahl-Hirschman Index

The further analysis starts by highlighting the features of the Herfindahl-Hirschman index, which for some may seem to be well-known or easy to understand intuitively. Given the number N of enterprises in the market, the *HHI* has the minimum value when all the enterprises have equal market shares. This statement is supported by the calculations.

Assume $\Delta d_i = d_i - \overline{d}$ to be the distance (difference) between the market share of the i-th enterprise and the average market share $\overline{d}$ of all N enterprises in the market. Then the formula of the *HHI* could be expressed as follows:

$$HHI = \sum_{i=1}^{N} d_i^2 = \left(\bar{d} + \Delta d_1\right)^2 + \left(\bar{d} + \Delta d_2\right)^2 + \ldots + \left(\bar{d} + \Delta d_N\right)^2 =$$
$$= \bar{d}^2 + 2\bar{d}\Delta d_1 + \left(\Delta d_1\right)^2 + \bar{d}^2 + 2\bar{d}\Delta d_2 + \left(\Delta d_2\right)^2 + \ldots + \bar{d}^2 + 2\bar{d}\Delta d_N + \left(\Delta d_N\right)^2 = \quad (3)$$
$$= N\bar{d}^2 + 2\bar{d}\sum_{i=1}^{N}\Delta d_i + \sum_{i=1}^{N}\left(\Delta d_i\right)^2.$$

It has to be noticed that the sum of the distances between the market share d_i and the average $\bar{d}$ is equal to zero ($\sum_{i=1}^{N}\Delta d_i = 0$), and that could be easily proved by the following calculation:

$$\sum_{i=1}^{N}\Delta d_i = \Delta d_1 + \Delta d_2 + \ldots + \Delta d_N = \left(d_1 - \bar{d}\right) + \left(d_2 - \bar{d}\right) + \ldots + \left(d_N - \bar{d}\right) =$$
$$= \sum_{i=1}^{N} d_i - N\bar{d} = \sum_{i=1}^{N} d_i - N\frac{\sum_{i=1}^{N} d_i}{N} = 0. \quad (4)$$

Now we can simplify Formula (3) to obtain the equation:

$$HHI = \sum_{i=1}^{N} d_i^2 = N\bar{d}^2 + \sum_{i=1}^{N}\left(\Delta d_i\right)^2 , \quad (5)$$

which has to fulfil the following conditions:

(1) $N\bar{d}^2 > 0$, because $N > 0$ and $\bar{d}^2 > 0$;

(2) $\sum_{i=1}^{N}\left(\Delta d_i\right)^2 \geq 0$, as we have the sum of squares, and for every *i*-th enterprise $\left(\Delta d_i\right)^2 \geq 0$. Moreover, $\sum_{i=1}^{N}\left(\Delta d_i\right)^2 = 0$ only in the case that $\Delta d_i = d_i - \bar{d} = 0 \ \left(\forall i \in [1; N]\right)$.

In the latter case (when $\sum_{i=1}^{N}\left(\Delta d_i\right)^2 = 0$), the following condition must be respected: $\Delta d_i = 0 \Rightarrow d_i = \bar{d} \quad \left(\forall i \in [1; N]\right)$, thus meaning that the *HHI*, calculated for a market of *N* enterprises, obtains the minimum value $N\bar{d}^2$ when the market shares of all *N* enterprises are equal. An example of four hypothetical markets is provided in Table 2 to clarify the theoretical analysis implemented: the market shares of all the players are equal in the first market, while in the other three markets they are not. The value of the *HHI* grows from 0.25 in market 1 to 0.52 in market 4, as the deviation of individual market shares increases from the average.

Table 2 Modelling the Changes in the Values of the *HHI* Depending on the Distribution of Market Shares

Market 1			Market 2			Market 3			Market 4		
Enterprise	Market share	*HHI*	Enterprise	Market share	*HHI*	Enterprise	Market share	*HHI*	Enterprise	Market share	*HHI*
1	0.25	0.25	1	0.3	0.26	1	0.4	0.34	1	0.7	0.52
2	0.25		2	0.3		2	0.4		2	0.1	
3	0.25		3	0.2		3	0.1		3	0.1	
4	0.25		4	0.2		4	0.1		4	0.1	

Source: Author.

If all N enterprises in the market have equal market shares, then the *HHI* would be minimized (for a number of enterprises N), and its formula would be:

$$\min HHI = N\bar{d}^2 = N\left(\frac{\sum_{i=1}^{N} d_i}{N}\right)^2 = \left(\sum_{i=1}^{N} d_i\right)^2 : N. \quad (6)$$

One can easily determine that the sum of all the enterprises' market shares is equal to 1 ($\sum_{i=1}^{N} d_i = 1$); thus, the minimum *HHI* can be written in a simpler form as $\min HHI = \frac{1}{N}$. However, Formula (6) is more important for us to take into account in the further analysis.

Continuing with the theoretical analysis, assume that there is a market of nine enterprises, in which the market shares of the four largest players are known (Table 3):

Table 3 Market Shares of Enterprises in a Hypothetical Market of Nine Players

Enterprise	1	2	3	4	5	6	7	8	9
Market share	0.3	0.25	0.2	0.15	?	?	?	?	?

Source: Author.

As the market shares of the fifth to ninth enterprises are not available, the *HHI* cannot be calculated. However, the following at least could be stated:

(a) the value of the *HHI* would be at least $0.3^2 + 0.25^2 + 0.2^2 + 0.15^2 = 0.215$;

(b) the value of the indicator would definitely exceed 0.215, because the squared market shares of the remaining five enterprises would be added. We do not know the exact contribution of the smaller enterprises to the value of the *HHI*, but it would be at least the sum of their squared market shares with the assumption that they are equal.

In this case the remaining five enterprises together control the market share of (1 – 0.3 – 0.25 – 0.2 – 0.15) = 0.1. With the assumption that their individual market shares are equal, each of them controls 0.1 : 5 = 0.02 of the total market. Then the value of *HHI* would exceed 0.215 by at least the amount of $0.02^2 + 0.02^2 + 0.02^2 + 0.02^2 + 0.02^2 = 0.002$. Therefore, in the analysed market, in which the market shares of the four largest enterprises only are known, the minimum value of the *HHI* would be 0.217. To clarify the analysis implemented, a number of possible scenarios of market shares' distribution in this market are presented in Table 4.

Table 4 Modelling the Scenarios of Market Shares' Distribution in a Nine-Enterprise Market in which the Market Shares of the Four Largest Enterprises Only Are Available

Scenario 1			Scenario 2			Scenario 3			Scenario 4		
Enterprise	Market share	*HHI*	Enterprise	Market share	*HHI*	Enterprise	Market share	*HHI*	Enterprise	Market share	*HHI*
1	**0.3**		**1**	**0.3**		**1**	**0.3**		**1**	**0.3**	
2	**0.25**		**2**	**0.25**		**2**	**0.25**		**2**	**0.25**	
3	**0.2**		**3**	**0.2**		**3**	**0.2**		**3**	**0.2**	
4	**0.15**		**4**	**0.15**		**4**	**0.15**		**4**	**0.15**	
5	0.02	0.217	5	0.03	0.21725	5	0.04	0.21755	5	0.09	0.22313
6	0.02		6	0.025		6	0.02		6	0.003	
7	0.02		7	0.02		7	0.01		7	0.003	
8	0.02		8	0.015		8	0.015		8	0.003	
9	0.02		9	0.01		9	0.015		9	0.001	

Source: Author.

The value of the *HHI* grows from Scenario 1 to Scenario 4 as the deviation of the market shares of the remaining five enterprises increases from their average of 0.02.

With regard to the theoretical analysis and scenario modelling implemented, it can be presumed that, for a market in which the market shares of the several largest enterprises are known, the minimum value of the *HHI* could be calculated, composed of two components (summands):

(i) The sum of the squared market shares of the enterprises, of which the actual market shares are available to the researcher. This summand itself represents the discrete indicator, the value of which depends on the total market share of the enterprises in question as well as the distribution of their individual market shares (thus having certain features that may seem common to both the CR_M and the *HHI*).

(ii) The sum of the squared average market shares of the remaining players for which the actual market shares are not available. This summand represents the minimum contribution of the remaining enterprises (of which the market shares are unknown) to the value of the *HHI*.

By dividing the set of *N* enterprises in the market into two intervals as written earlier, we obtain the following *HHI* formula:

$$HHI = \sum_{i=1}^{M} d_i^2 + \sum_{i=M+1}^{N} d_i^2, \tag{7}$$

where $i \in [1; M]$ is the first interval for which the market shares d_i of the enterprises are available and $i \in [M+1; N]$ is the second interval for which the market shares are unknown but assumed to be equal.

From Formulas (6) and (7), we can draw the formula of the minimum *HHI*:

$$\min HHI_M = \sum_{i=1}^{M} d_i^2 + (1 - \sum_{i=1}^{M} d_i)^2 : (N - M), \tag{8}$$

with respect to condition:

$$\min \sum_{i=M+1}^{N} d_i^2 = \left(\frac{1 - \sum_{i=1}^{M} d_i}{N - M} \right)^2 \cdot \left(N - M\right) = (1 - \sum_{i=1}^{M} d_i)^2 : (N - M), \tag{9}$$

where: $1 - \sum_{i=1}^{M} d_i$ is the total market share controlled by the second-interval enterprises (for which the individual market shares d_i are unknown); *M* is the number of enterprises in the first interval (for which the individual market shares d_i are available); *N* is the total number of enterprises in the market; and $N - M$ is the number of enterprises in the second interval.

In Formula (9) the value of $\sum_{i=M+1}^{N} d_i^2$ is minimized at $(1 - \sum_{i=1}^{M} d_i)^2 : (N - M)$ in the case that the market shares d_i of all the enterprises in the second interval $i \in \left[M + 1; N\right]$ are equal and could be expressed as their average $d_i = \overline{d} = \left(\frac{1 - \sum_{i=1}^{M} d_i}{N - M} \right)$.

That is also supported by the minimum value of the *HHI* obtained in Formula (6), as what is valid for *N* enterprises (the whole market) is also valid for the number of enterprises $N - M$ (the second interval).

In the author's opinion, the minimum Herfindahl-Hirschman index (minHHI_M) should be applied to concentrated markets only: calculated on the basis of the market shares of the largest enterprises, it would not diverge significantly from the actual value of the *HHI*, since the strongest impact on its value comes from the largest players while the sales volumes of smaller enterprises would not significantly affect its value. To support this statement, the modelling of hypothetical markets is implemented in Section 3 of the paper.

The application of the minimum Herfindahl-Hirschman index is based on practical grounds – it would allow the measurement of concentration and uneven distribution of market shares accurately enough while accounting for the sales volumes of the largest companies only. More arguments in favour of the indicator follow:

(1) The relationship between the *HHI* and the $\min HHI_M$ in the same market is always $HHI \geq \min HHI_M$. One might argue that it does not look like an advantage and rather reflects the problem of the estimation accuracy. However, in concentrated markets it could be enough to draw a conclusion on the competitive environment: for

example, in the case that the value of $\min HHI_M$, calculated on the basis of *M* largest companies' market shares, exceeds 0.180, then certainly the actual value of $HHI > 0.180$ and the market could be considered as highly concentrated.

(2) The minimum Herfindahl-Hirschman index does not absolutely ignore the enterprises for which the market shares are not available for research. On the contrary, the value of the $\min HHI_M$ is sensitive to the total number of enterprises in the second interval ($i \in [M+1; N]$) – a growing number of companies decreases the value of the indicator, thus reflecting more competitive conditions in the market.

(3) The application of the $\min HHI_M$ could easily supplement the analysis of concentration based on the concentration ratio (CR_M) only (that is usually the case when the data are limited) and provide more information on the distribution of market shares between *M* largest enterprises and overall market concentration without requiring any additional data on market shares or sales volumes to be collected (for empirical evidence, see Ginevičius and Krivka 2009; Krivka 2010).

To summarize this section of the paper, its key points are indicated. The value of the *HHI* is minimized for a market of *N* enterprises in the case that all of them control equal market shares – that becomes the basis for the $\min HHI_M$, calculated for a market in which the market shares of *M* largest companies are available while the market shares of the remaining enterprises are unknown and thus assumed to be equal. The proposed indicator is supposed to be applied to concentrated markets, while its application is based on practical grounds related to research data availability.

3. Modelling the Application of the Minimum Herfindahl-Hirschman Index

In this section the opinion that in concentrated markets the value of the $\min HHI_M$ (see Formula (8)) would not significantly deviate from the actual value of the *HHI* (see Formula (2)) is supported by modelling hypothetical markets.

In the first scenario each market has three large oligopolic enterprises, of which the total market share varies from 60 to 90 per cent, and ten smaller enterprises (from 4 to 13), of which the market shares decrease in geometric progression (Table 5). To calculate the values of the $\min HHI_M$, it is assumed that the market shares of the three largest companies only are available, while those of enterprises 4 to 13 are unknown and thus assumed to be equal.

In terms of the two intervals of enterprises in a market, indicated in Section 2, the first interval $i \in [1; M]$, for which the market shares d_i are available, consists of three companies ($M = 3$), while the second one $i \in [M+1; N]$, for which the market shares are unknown, totals ten enterprises ($N - M = 10$). The calculated values of the $\min HHI_3$ and *HHI* are then compared by absolute and relative deviations.

Table 5 Modelling the Deviations between the Values of min*HHI*$_3$ and *HHI* based on Geometric Progression

Enterprises	Market shares of the enterprises (in hypothetical markets 1-7)						
	1	**2**	**3**	**4**	**5**	**6**	**7**
1	0.4000	0.3000	0.3000	0.3000	0.3000	0.2000	0.2000
2	0.3000	0.3000	0.3000	0.2000	0.2000	0.2000	0.2000
3	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
The total market share of oligopolic enterprises (1-3)	**0.9000**	**0.8000**	**0.8000**	**0.7000**	**0.7000**	**0.6000**	**0.6000**
4	0.0500	0.1000	0.0500	0.1000	0.0500	0.1000	0.0500
5	0.0250	0.0500	0.0384	0.0673	0.0440	0.0768	0.0475
6	0.0125	0.0250	0.0295	0.0453	0.0387	0.0590	0.0451
7	0.0063	0.0125	0.0226	0.0305	0.0340	0.0453	0.0428
8	0.0031	0.0063	0.0174	0.0205	0.0299	0.0348	0.0406
9	0.0016	0.0031	0.0133	0.0138	0.0263	0.0267	0.0385
10	0.0008	0.0016	0.0102	0.0093	0.0231	0.0205	0.0366
11	0.0004	0.0008	0.0079	0.0063	0.0203	0.0157	0.0347
12	0.0002	0.0004	0.0060	0.0042	0.0179	0.0121	0.0330
13	0.0001	0.0002	0.0046	0.0028	0.0157	0.0093	0.0313
The total market share of the remaining enterprises (4-13)	**0.1000**	**0.2000**	**0.2000**	**0.3000**	**0.3000**	**0.4000**	**0.4000**
The contribution of oligopolic enterprises' market shares to the value of the *HHI*	0.2900	0.2200	0.2200	0.1700	0.1700	0.1200	0.1200
The contribution of the remaining enterprises' market shares to the value of the *HHI*	0.0033	0.0133	0.0061	0.0183	0.0102	0.0242	0.0164
HHI	0.2933	0.2333	0.2261	0.1883	0.1802	0.1442	0.1364
min*HHI*$_3$	0.2910	0.2240	0.2240	0.1790	0.1790	0.1360	0.1360
Absolute deviation, *HHI*-min*HHI*$_3$	**0.0023**	**0.0093**	**0.0021**	**0.0093**	**0.0012**	**0.0082**	**0.0004**
Relative deviation, (*HHI*-min*HHI*$_3$): *HHI*	**0.80 %**	**4.00 %**	**0.91 %**	**4.93 %**	**0.66 %**	**5.71 %**	**0.26 %**

Source: Author.

The implemented modelling supports the idea that the impact of the ten remaining smaller enterprises on the value of the *HHI* is insignificant in the case that the largest of the remaining enterprises (4-13) in fact controls up to 5 per cent of the market. In that case the relative deviation between the values of $\min HHI_3$ and *HHI* is less than 1 per cent; thus, the $\min HHI_3$ seems to be accurate enough and could be treated as an easy empirical application measure of concentration. In the modelled markets, in the case that the largest of the remaining enterprises has a market share of 10 per cent, the relative deviation is in the range of 4-6 per cent; thus, possible errors should be taken into account.

Another modelling scenario is applied in Table 6, in which the main assumptions are the same (three large oligopolic enterprises of which the total market share varies from 60 to 90 per cent, plus ten smaller enterprises), except for the market shares in the second interval – in this case they are absolutely random. To obtain random numbers, the function RANDBETWEEN of MS Excel was applied to the interval from 1 to 100; then the numbers obtained were normalized with respect to the total market share of the remaining enterprises in each market.

Table 6 Modelling the Deviations between the Values of minHHI_3 and HHI Based on Random Numbers

Enterprises	Market shares of the enterprises (in hypothetical markets 1-12)											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	0.4000	0.4000	0.4000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.2000	0.2000	0.2000
2	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
3	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
The total market share of oligopolic enterprises (1-3)	**0.9000**	**0.9000**	**0.9000**	**0.8000**	**0.8000**	**0.8000**	**0.7000**	**0.7000**	**0.7000**	**0.6000**	**0.6000**	**0.6000**
4	0.0101	0.0053	0.0145	0.0279	0.0290	0.0087	0.0605	0.0390	0.0279	0.0199	0.0691	0.0261
5	0.0129	0.0127	0.0015	0.0173	0.0374	0.0049	0.0455	0.0047	0.0385	0.1062	0.0660	0.0308
6	0.0102	0.0112	0.0105	0.0272	0.0174	0.0340	0.0468	0.0163	0.0408	0.0133	0.0746	0.0615
7	0.0098	0.0008	0.0065	0.0145	0.0062	0.0393	0.0119	0.0026	0.0346	0.0581	0.0093	0.0207
8	0.0053	0.0070	0.0103	0.0046	0.0196	0.0413	0.0331	0.0390	0.0380	0.0066	0.0194	0.0515
9	0.0073	0.0080	0.0185	0.0332	0.0036	0.0083	0.0193	0.0385	0.0536	0.0498	0.0474	0.0368
10	0.0116	0.0129	0.0105	0.0339	0.0303	0.0073	0.0050	0.0422	0.0050	0.0581	0.0124	0.0435
11	0.0116	0.0162	0.0078	0.0102	0.0236	0.0272	0.0480	0.0501	0.0028	0.0315	0.0311	0.0622
12	0.0129	0.0127	0.0141	0.0039	0.0223	0.0073	0.0094	0.0200	0.0302	0.0266	0.0186	0.0602
13	0.0082	0.0131	0.0059	0.0272	0.0107	0.0218	0.0206	0.0475	0.0285	0.0299	0.0520	0.0067
The total market share of the remaining enterprises (4-13)	**0.1000**	**0.1000**	**0.1000**	**0.2000**	**0.2000**	**0.2000**	**0.3000**	**0.3000**	**0.3000**	**0.4000**	**0.4000**	**0.4000**
The contribution of oligopolic enterprises' market shares to the value of the *HHI*	0.2900	0.2900	0.2900	0.2200	0.2200	0.2200	0.1700	0.1700	0.1700	0.1200	0.1200	0.1200
The contribution of the remaining enterprises' market shares to the value of the *HHI*	0.0011	0.0012	0.0012	0.0052	0.0051	0.0059	0.0124	0.0118	0.0112	0.0237	0.0216	0.0193
HHI	0.2911	0.2912	0.2912	0.2252	0.2251	0.2259	0.1824	0.1818	0.1812	0.1437	0.1416	0.1393
minHHI_3	0.2910	0.2910	0.2910	0.2240	0.2240	0.2240	0.1790	0.1790	0.1790	0.1360	0.1360	0.1360
Absolute deviation, *HHI*-minHHI_3	**0.0001**	**0.0002**	**0.0002**	**0.0012**	**0.0011**	**0.0019**	**0.0034**	**0.0028**	**0.0022**	**0.0077**	**0.0056**	**0.0033**
Relative deviation, (*HHI*-minHHI_3): *HHI*	**0.02%**	**0.07%**	**0.07%**	**0.52%**	**0.47%**	**0.84%**	**1.85%**	**1.52%**	**1.22%**	**5.37%**	**3.94%**	**2.35%**

Source: Author.

The modelling based on random numbers also supports the idea of insignificant deviations between the values of $\min HHI_3$ and HHI in concentrated markets. Particularly, in the case that the companies with available market shares total from 80 to 90 per cent, the relative deviation is just under 1 per cent; for the total market share of the first-interval companies of 70 per cent, the relative deviation is between 1 and 2 per cent. Only in the case of a 60 per cent market share controlled by the largest companies do we obtain a deviation between the values of $\min HHI_3$ and HHI in the range of 2-6 per cent, which may indicate some limitations in the results of the empirical research.

Furthermore, for the analysis based on random numbers not to seem accidental, we extend it to 100 tries for each amount of the total market share of the largest companies (60, 70, 80, and 90 per cent), keeping the rest of the modelling assumptions the same as in Table 6, and calculate the average relative deviation between the values of $\min HHI_3$ and HHI, coupled by the distribution of the deviation values among three intervals: under 1 per cent, between 1 and 2 per cent, and more than 2 per cent (Table 7).

Table 7 Summary Results of the Extended Modelling of the Deviations between the Values of minHHI_3 and HHI based on Random Numbers

Total market share of oligopolic enteprises (1-3)	Average relative deviation	Number of tries			
		Total	With relative deviation under 1%	With relative deviation between 1% and 2%	With relative deviation more than 2%
90%	0.11%	100	100	0	0
80%	0.62%	100	88	12	0
70%	1.62%	100	23	51	26
60%	3.62%	100	1	13	86

Source: Author.

The results of the extended modelling based on random numbers in general support the opinion that the minHHI_M is applicable to concentrated markets, for its value deviates insignificantly from the actual value of the HHI in the case that the total market share controlled by the largest companies is between 70 and 90 per cent. However, larger deviation is expected in less concentrated markets with less than a 70 per cent market share of the total oligopolic enterprises.

4. Conclusions

Concentration has become one of the main measures quantitatively characterizing the market structure in recent empirical research in the context of the most popular industrial organization approaches: the structure-conduct-performance approach and the efficiency hypothesis. A number of concentration indicators, based on the concept of the concentration curve, are proposed in theoretical sources on the subject; however, the concentration ratio and Herfindahl-Hirschman index are the most widely applied in empirical research, which was supported by the brief overview of recent empirical studies provided in this paper.

The main problem of the practical application of the HHI is the necessity to incorporate the market shares of all the enterprises in the market. As an alternative, the minimum Herfindahl-Hirschman index, calculated by splitting the set of enterprises in the market into two intervals – one containing the largest companies of which the market shares are available for research and the other one containing the remaining enterprises with unknown market shares that are assumed to be equal – was analysed in the paper from both the theoretical and the practical point of view, modelling different scenarios of market shares' distribution in hypothetical markets.

It was determined that the minHHI_M could be applied to concentrated markets only: calculated on the basis of the market shares of the largest enterprises, it would not significantly diverge from the actual values of the HHI, because the strongest impact on its value comes from the largest players while the sales volumes of the smaller enterprises would not significantly affect its value. However, possible differences between the values of minHHI_M and the actual values of HHI should be considered by the researcher, especially in the case that one or more larger enterprises fall out of the first interval; that is, their market shares are unknown.

Though the minHHI_M ignores the actual market shares of smaller enterprises, it has to be stressed that the indicator is sensitive to the total number of enterprises in the market – a growing number of the remaining (second-interval) enterprises decreases the value of the indicator, thus reflecting more competitive conditions in the market.

From the practical point of view, the minHHI_M could supplement the analysis of concentration based on the concentration ratio only (in the case that the data are limited) and provide more information on the possibly uneven distribution of market shares between the largest enterprises and the overall market concentration without requiring any additional data on market shares or sales volumes to be collected.

References

Akomea, Samuel Y., and Michael Adusei. 2013. "Bank Recapitalization and Market Concentration in Ghana's Banking Industry: A Herfindahl-Hirschman Index Analysis." *Global Journal of Business Research*, 7(3): 31-45.

Bain, Joe S. 1956. *Barriers to New Competition: Their Character and Consequences in Manufacturing Industries*. Cambridge, MA: Harvard University Press.

Bain, Joe S. 1959. *Industrial Organization*. New York: John Wiley.

Beattie, Vivien, Alan Goodacre, and Stella Fearnley. 2003. "And Then There Were Four: A Study of UK Audit Market Concentration - Causes, Consequences and the Scope for Market Adjustment." *Journal of Financial Regulation and Compliance*, 11(3): 250-265.

Beck, Jason, Frank Scott, and Aaron Yelowitz. 2012. "Concentration and Market Structure in Local Real Estate Markets." *Real Estate Economics*, 40(3): 422-460.

Bobillo, Alfredo M., Miguel A. F. Temprano, and Fernando T. Gaite. 2004. "The Dynamics of Concentration and Inequality in Spanish Industries." *Management Research News*, 27(8/9): 57-75.

Čepinskis, Jonas, and Kristina Gancevskaitė. 2008. "Qualitative Evaluation of Concentration and Competition in Lithuanian Life Insurance Market." *Applied Economics: Systematic Research*, 2(1): 99-112.

Darškuvienė, Valdonė, and Jonė Šakalytė. 2009. "Efficiency of Stock Exchange Network Formation as a Change in Market Concentration." *Applied Economics: Systematic Research*, 3(1): 63-77.

Deltuvaitė, Vilma, and Vilda Gižienė. 2007. "Competition, Concentration, Efficiency, and Their Relationship in Lithuanian Banking Sector." *Economics and Management*, 12: 188-201.

Deltuvaitė, Vilma. 2009. "The Research of Concentration Effect on Banking System Financial Stability." *Economics and Management*, 14: 156-165.

Demsetz, Harold. 1973. "Industry Structure, Market Rivalry and Public Policy." *Journal of Law and Economics*, 16(1): 1-10.

Ginevičius, Romualdas, and Stasys Čirba. 2007. "Determining Market Concentration." *Journal of Business Economics and Management*, 8(1): 3-10.

Ginevičius, Romualdas, and Stasys Čirba. 2009. "Additive Measurement of Market Concentration." *Journal of Business Economics and Management*, 10(3): 191-198.

Ginevičius, Romualdas, and Algirdas Krivka. 2009. "The Research on Business Concentration in Lithuanian Economy." *Business: Theory and Practice*, 10(3): 191-203.

Ginevičius, Romualdas, Vladislavas Petraškevičius, and Jolita Šimkūnaitė. 2010. "The Influence of Market Concentration on the Results of Enterprise Commercial Activities." *Business: Theory and Practice*, 11(3): 185-193.

Gini, Corrado. 1936. "On the Measure of Concentration with Special Reference to Income and Statistics." *Colorado College Publication, General Series*, 208: 73-79.

Hani, Peter K. 1987. *Die Messung der Unternehmenskonzentration: Eine Theoretische und empirische Evaluation von Konzentrationsmaßen*. Grüsch: Rüegger.

Hays, Fred H., Stephen A. DeLurgio, and Arthur H. Gilbert Jr. 2009. "Concentration, the Internet and Pricing of Bank Assets and Liabilities." *Research in Business and Economics Journal*, 1: 1-13.

Horowitz, Ann, and Ira Horowitz. 1968. "Entropy, Markov Processes and Competition in the Brewing Industry." *Journal of Industrial Economics*, 16: 196-211.

Hoxha, Indrit. 2013. "The Effect of Banking Market Structure on the Volatility of Growth of Manufacturing Sectors in Developing Countries." *Journal of Economics and Finance*, 37(4): 528-546.

Kirzner, Israel M. 1963. *Price System*. New Jersey: D. Van Norstrand Company.

Krivka, Algirdas. 2010. "The Elements of Competitive Environment of an Enterprise: A Case of Oligopolic Markets Comparative Analysis." *Science - Future of Lithuania*, 2(2): 32-37.

Liu, Li, and Rosanne Altshuler. 2013. "Measuring the Burden of the Corporate Income Tax under Imperfect Competition." *National Tax Journal*, 66(1): 215-238.

Liu, Charles Z., Chris F. Kemerer, Sandra A. Slaughter, and Michael D. Smith. 2012. "Standards Competition in the Presence of Digital Conversion Technology: An Empirical Analysis of the Flash Memory Card Market." *MIS Quarterly*, 36(3): 921-942.

Marginean, Silvia, and Ramona Toma. 2011. "Some Issues Concerning Sector Concentration. Evidence from Romania." *Journal of Business Economics and Management*, 12(1): 110-130.

Mariscal, Judith. 2009. "Market Structure and Penetration in the Latin American Mobile Sector." *Info*, 11(2): 24-41.

Mensi, Sami. 2010. "Measurement of Competitiveness Degree in Tunisian Deposit Banks: An Application of the Panzar and Rosse Model." *Panoeconomicus*, 57(2): 189-207.

Moretti, Luigi. 2012. "Bank Concentration, Private Credit, and Firm Turnover Evidence from the Enlarged European Union." *Eastern European Economics*, 50(5): 5-22.

Mukhopadhyaya, Jayanta N., Malabika Roy, and Ajitava Raychaudhuri. 2012. "Change in Concentration and Inequality after Liberalization in the Cement Industry of India." *XIMB Journal of Management*, 9(2): 1-26.

Nguyen, Thao N., and Chris Stewart. 2013. "Concentration and Efficiency in the Vietnamese Banking System between 1999 and 2009. A Structural Model Approach." *Journal of Financial Regulation and Compliance*, 21(3): 268-283.

Peltzman, Sam. 1977. "The Gains and Losses from Industrial Concentration." *Journal of Law and Economics*, 20(2): 229-263.

Ponikvar, Nina, and Maks Tajnikar. 2011. "Are the Determinants of Markup Size Industry-Specific? The Case of Slovenian Manufacturing Firms." *Panoeconomicus*, 58(2): 229-244.

Raghavan, Vedapuri S. 2013. "Consolidation in the Airline Industry: The Delta-Northwest Merger." *Journal of International Finance and Economics*, 13(2): 89-96.

Rosenbluth, Gideon. 1961. "Diskussionsbeitrag." In *Round Table-Gespräch über Messung des industriellen Konzentration*, ed. J. Nichans, 367-395. Berlin: Duncker & Humbolt.

Sandulli, Francesco D., Jose Fernandez-Menendez, Antonio Rodriguez-Duarte, and Jose Ignacio Lopez-Sanchez. 2012. "Testing the Schumpeterian Hypotheses on an Open Innovation Framework." *Management Decision*, 50(7): 1222-1232.

Shapiro, Milton M. 1985. *Foundations of the Market-Price System*. Boston: University Press of America.

Von Mises, Ludwig. 1996. *Human Action: A Treatise on Economics*. 4th revised ed. San Francisco: Fox & Wilkes.

Von Mises, Ludwig. 1998. "Monopoly Prices." *Quarterly Journal of Austrian Economics*, 1(2): 1-28.