

Modelling Seasonality – An Extension of the HEGY Approach in the Presence of Two Structural Breaks

Ozlem Tasseven*

Summary: In this paper the HEGY testing procedure (Hylleberg et al. 1990) of analysing seasonal unit roots is tried to be re-examined by allowing for seasonal mean shifts with exogenous break points. Using some Monte Carlo experiments the distribution of the HEGY and the extended HEGY tests for seasonal unit roots subject to mean shifts and the small sample behavior of the test statistics have been investigated. Based on an empirical analysis upon the conventional money demand relationships in the Turkish economy, our results indicate that seasonal unit roots appear for the GDP deflator, real M2 and the expected inflation variables while seasonal unit roots at annual frequency seem to be disappear for the real M1 balances when the possible structural changes in one or more seasons at 1994 and 2001 crisis years have been taken into account.

Key words: HEGY Seasonal unit root test, Deterministic seasonality, Structural breaks, Money demand, Turkish economy

JEL: C01, C15, C51, C88, E41

Introduction

The study of the seasonal properties of the economic time series has been of a special interest for both academicians and researchers in the last decades. Following the pioneering paper of Hylleberg et al. (1990) (henceforth HEGY) many other papers examine this issue and extensively reveal that most seasonal time series data have been subject to unit roots at seasonal frequencies rather than being subject to non-stationarity at the zero frequency (see, e.g. Miron, 1994; and Ghysels, 1994; among others). They show that some variables have a deterministic seasonal pattern while others tend to be characterized by seasonal movements that change slowly over time. In this sense, to examine whether the seasonality in the time series considered must be best described by a determinis-

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tic process or by stochastic trends at seasonal frequencies constitutes a new research area to be analyzed and to be empirically tested.

Perron (1989, 1990) in his seminal papers assert that the conventional unit root tests such as the most conventional one proposed by Dickey and Fuller (1979) do not consider that a possible known structural break in the trend function may tend too often not to reject the null hypothesis of a unit root in the time series when in fact the series is stationary around a one time structural break. Perron (1989) and Lopes and Montanes (2005) argue that such an issue can also be extended for seasonal unit roots and seasonal mean shifts and that neglected (seasonal) mean shifts can likely to be biased towards non-rejection of the unit roots. When some recent literature has been examined we can see that modelling seasonality in relation to the HEGY seasonal unit root testing with one structural break is conducted by investigation of a break point. Papers such as Zivot and Andrews (1992), Smith and Otero (1997), Franses and Vogelsang (1995) and Franses and Hobijn (1997) extend the HEGY procedure in implementing the unit root testing by allowing for a known breakpoint, while Franses and Vogelsang (1998), Balcombe (1999), Harvey et al. (2001) and a recent paper by Popp (2007) try to consider the case of an unknown breakpoint to be recursively estimated from the data.

In this paper, our aim is to extend the HEGY testing procedure by allowing for the seasonal mean shifts in more than one year while considering also exogenous break points. Franses and Vogelsang (1995) analyse additive and innovational outlier tests for seasonal unit roots in the presence of seasonal mean shifts to occur in one year. Following the study of Franses and Vogelsang (1995) our contribution to this literature is to somewhat generalize their approach and hence to apply to double break points and then generate the relevant critical values. For this purpose, some innovational outlier tests for seasonal unit roots have been tried to be analysed. The organization of the paper is as follows. Section two introduces fundamental building blocks of the HEGY testing procedure, while section three presents a detailed overview of the seasonal unit root tests with seasonal mean shifts allowed for one year. Section four deals with a modified version of the HEGY estimation procedure in the presence of seasonal mean shifts in two years. Such an approach developed in this paper takes account of not only seasonal unit root tests but also of the effects of shocks to the system such as policy interventions or other shocks or crises which are expected to have a considerable impact upon current domestic macroeconomic developments. In section five some bases of the Monte Carlo simulation experiments conducted in this paper have been highlighted. The critical values so created can be used in modelling more than one structural break that affect the trend and development of the economy in the context of seasonal unit root testing. In section six an empirical money demand model using the HEGY testing procedures is conducted for the Turkish economy. The last section summarizes results and concludes.

1. Seasonal unit root tests procedures

A large body of seasonal unit root tests has been proposed to test for the appropriateness of the filters Δ_1 and Δ_s for removing non-seasonal and seasonal stochastic trends in the time series data. In this sense, the most important seasonal unit root tests can be attributed to the estimation procedures developed by Dickey et al. (1984), Osborn et al. (1988), HEGY (1990) and Canova and Hansen (1995). Among all these HEGY (1990) is the one widely used to test for seasonal and non-seasonal unit roots in a univariate series. This can be shown based on the following auxiliary regression:

$$\varphi(L)y_{4,t} = \mu_t + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \varepsilon_t \tag{1}$$

where $\varphi(L)$ is an AR polynomial of order $p-4$ and ε_t a normally and independently distributed (i.i.d) error term with zero mean and constant variance. μ_t is defined as:

$$\mu_t = \alpha + \sum_{s=1}^3 \delta_s D_{st} + \beta_t \tag{2}$$

where:

$$\begin{aligned} y_{1,t} &= (1+L+L^2+L^3) y_t \\ y_{2,t} &= -(1-L)(1+L^2) y_t \\ y_{3,t} &= -(1-L^2) y_t \\ y_{4,t} &= (1-L^4) y_t \end{aligned}$$

The polynomial $(1 - L^4)$ can be expressed as $(1 - [L^4]) = (1 - L) (1 + L) (1 - iL) (1 + iL) = (1 - L) (1 + L) (1 + L^2)$ with the two complex roots given as $(1 - iL)(1 + iL) = (1 + L^2)$. Deterministic components which include an intercept (α), three seasonal dummies (D_{st}) and a time trend (β_t) are also included in equation (1) that can be estimated by ordinary least squares (OLS) estimators. The relevant null and alternative hypotheses to be tested can be given as follows:

$$[H_0: \pi_1 = 0,] [H_1: \pi_1 < 0]; \tag{3}$$

$$[H_0: \pi_2 = 0], [H_1: \pi_2 < 0]; \tag{4}$$

$$[H_0: \pi_3 = 0], [H_0: \pi_4 = 0], [H_1]: [\pi_3 \neq 0 \text{ or } \pi_4 \neq 0] \tag{5}$$

The HEGY test involves the use of the t -test for the first two hypotheses and an F -test for the third hypothesis. Non-rejection of the first hypothesis would mean a unit root at the zero frequency or a non-seasonal unit root in the series. Non-

rejection of the second hypothesis would show that there exists seasonal unit root at the semi-annual frequency. Finally, if the third hypothesis is not rejected we can infer that there exists a seasonal unit root at the annual frequency. These null hypotheses are tested separately. Critical values for the one sided t -tests for π_1 to π_4 (F_{34}) have been given in HEGY (1990).

2. Seasonal unit root tests with seasonal mean shifts in one year

Distorting observations in the time series data which are called outliers can be observed in a sequence or only as a single observation. Especially the case of single observation can be considered to have a major effect on time series modelling and forecasting. In this sense, outliers in differences may cause level shifts in the level series. Perron (1989) argues that structural changes to the trend function can be viewed as some kind of big shocks or infrequent events that have permanent effects on the level of the series. It is essential to take into consideration the way these big shocks affect the level of the variables, i.e. the way the transition to a new trend path occurs. As Perron (1990) suggested, in a long time series data, there would always be the possibility of the presence of multiple changes over time. Deterministic shifts in seasonal constants are likely to be occurred because of a change in the measurement techniques or of important economic events. Furthermore, such shifts may have an immediate or a gradual effect. For instance, the additive outlier model in Perron (1989) treats the shifts as immediate while the innovative outlier model approaches to the shifts as gradual.

Franses and Vogelsang (1995) try to consider testing for the seasonal unit roots in the presence of changing seasonal means with exogenous break point. It is assumed that there is a single break which occurs at time TB' where $1 < TB' < T$. The additive outlier model for quarterly time series under the null hypothesis of one non-seasonal unit root and three seasonal unit roots can be written as follows:

$$y_t = \sum_{s=1}^4 \kappa_s D(TB')_{s,t} + y_{t-4} + w_t \tag{6}$$

where:

- $D(TB')_{1,t} = 1$ if $t = TB_1 + 1$ (and zero elsewhere)
- $D(TB')_{2,t} = 1$ if $t = TB_1 + 2$ (and zero elsewhere)
- $D(TB')_{3,t} = 1$ if $t = TB_1 + 3$ (and zero elsewhere)
- $D(TB')_{4,t} = 1$ if $t = TB_1 + 4$ (and zero elsewhere)

Above w_t represents a stationary and invertible ARMA(p,q) process. Under the alternative hypothesis, the series y_t does not contain any of these unit roots and can be written as follows:

$$y_t = \sum_{s=1}^4 \mu_s D_{s,t} + \sum_{s=1}^4 \kappa_s DU_{s,t} + v_t \tag{7}$$

where v_t is a stationary and invertible ARMA($p+4,q$) process and $D_{s,t}$ the seasonal dummies for the entire sample for:

$$\begin{aligned} DU_{1,t} &= 1 \text{ if } t > TB' \text{ and } tmod=1, \text{ (and zero elsewhere)} \\ DU_{2,t} &= 1 \text{ if } t > TB' \text{ and } tmod=2, \text{ (and zero elsewhere)} \\ DU_{3,t} &= 1 \text{ if } t > TB' \text{ and } tmod=3, \text{ (and zero elsewhere)} \\ DU_{4,t} &= 1 \text{ if } t > TB' \text{ and } tmod=4, \text{ (and zero elsewhere)} \end{aligned}$$

Above $DU_{s,t} = 1 (t > TB')D_{s,t}$ where $1(\cdot)$ is the indicator function and $tmod$ shows the corresponding season of this function. $DU_{s,t}$ can be defined as seasonal dummies that only take non-zero values in the corresponding seasons when $t > TB'$. DU terms allow for the break under the alternative hypothesis.

Testing for seasonal unit roots using the additive outlier (AO) model can be implemented in two steps. First, the deterministic part must be removed from the series y_t :

$$y_t = \sum_{s=1}^4 \gamma_s D_{s,t} + \sum_{s=1}^4 \delta_s DU_{s,t} + \tilde{y}_t \tag{8}$$

Then the auxiliary regression using the residuals from the Eq. (8) must be considered:

$$\begin{aligned} \Delta_4 \tilde{y}_t &= \pi_1 \tilde{y}_{1,t-1} + \pi_2 \tilde{y}_{2,t-1} + \pi_3 \tilde{y}_{3,t-2} + \pi_4 \tilde{y}_{3,t-1} + \tilde{y}_{t-j} + \sum_{j=1}^4 c_j \Delta^4 \tilde{y}_{t-j} \\ &+ \sum_{s=1}^4 \varrho_s D(TB')_{s,t} + \sum_{s=1}^4 \eta_i D(TB')_{4,t-i} + \varepsilon_t \end{aligned} \tag{9}$$

where:

$$\begin{aligned} \tilde{y}_{1t} &= (1+B+B^2+B^3)\tilde{y}_t \\ \tilde{y}_{2t} &= -(1-B+B^2-B^3)\tilde{y}_t \\ \tilde{y}_{3t} &= -(1-B^2)\tilde{y}_t \end{aligned}$$

It can be observed from Eq. (6) and Eq. (7) that the AO model allows all seasonal constant to change. Therefore, the mean of the series may change, which affects testing for the non-seasonal unit root 1. In order to model the mean as a constant it is assumed that $\delta_1 + \delta_2 + \delta_3 + \delta_4 = 0$ in Eq. (8) above. This can be obtained by considering seasonal dummy variables $DU^*_{s,t}$ defined as follows:

$$\begin{aligned} DU^*_{1,t} &= DU_{1,t} - DU_{2,t} \\ DU^*_{2,t} &= DU_{2,t} - DU_{3,t} \end{aligned}$$

$$DU^*_{3,t} = DU_{3,t} - DU_{4,t}$$

On the other hand, the innovative outlier test is based on a single regression below:

$$\Delta_4 y_t = \mu_t + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \sum_{s=1}^4 \mu_s D_{s,t} + \sum_{s=1}^4 \delta_s DU_{s,t} + \sum_{s=1}^4 \kappa_s D(TB')_{s,t} + \gamma t + \varepsilon_t \tag{10}$$

Franses and Vogelsang (1995) present asymptotic and small sample critical values for additive and innovative outlier models for known and estimated break points. Franses and Vogelsang (1998) also consider an unknown break date for both the additive and innovative outlier models. In the additive outlier representation, under the null hypothesis one non-seasonal and three seasonal unit roots can be given as follows:

$$y_t = \sum_{s=1}^4 \kappa_s D(TB')_{s,t} + y_{t-4} + w_t \tag{11}$$

where $D(TB')_{s,t} = \Delta_4 DU_{s,t}$. Under the alternative hypothesis y_t does not contain these unit roots and can be written as follows:

$$y_t = \sum_{s=1}^4 \mu_s D_{s,t} + \sum_{s=1}^4 \kappa_s DU_{s,t} + v_t \tag{12}$$

where v_t is a stationary and invertible $ARMA(p+4, q)$ process. As in Franses and Vogelsang (1995) two stage procedure is implemented in order to test for seasonal unit roots, which incorporates additive outlier component. The deterministic part must initially be removed from y_t series:

$$y_t = \sum_{s=1}^4 \gamma_s D_{s,t} + \sum_{s=1}^4 \delta_s DU_{s,t} + \tilde{y}_t \tag{13}$$

In the second step auxiliary regression for \tilde{y}_t is considered:

$$\Delta_4 \tilde{y}_t = \pi_1 \tilde{y}_{1,t-1} + \pi_2 \tilde{y}_{2,t-1} + \pi_3 \tilde{y}_{3,t-2} + \pi_4 \tilde{y}_{3,t-1} + \sum_{j=1}^4 c_j \Delta_4 \tilde{y}_{t-j} + \sum_{s=1}^4 \vartheta_s D(TB')_{s,t} + \sum_{s=1}^4 \eta_i D(TB')_{4,t-i} + \varepsilon_t \tag{14}$$

where:

$$\tilde{y}_{1t} = (1+B+B^2+B^3)\tilde{y}_t \tag{15}$$

$$\tilde{y}_{2t} = -(1-B+B^2-B^3)\tilde{y}_t \tag{16}$$

$$\tilde{y}_{3t} = -(1-B^2)\tilde{y}_t \tag{17}$$

The test statistics of interest are $t\pi_1$, $t\pi_2$ and F_{34} in Eq. (14). In the innovative outlier model, seasonal mean shifts are incorporated where the shifts in seasonal means have a gradual effect on y_t . The null hypothesis of one non-seasonal unit root and three seasonal unit roots can be given as follows:

$$y_t = y_{t-4} + \psi(B)[u_t + \sum_{s=1}^4 \kappa_s D(TB')_{s,t}] \tag{18}$$

where u_t is i.i.d. $(0, \sigma^2)$ and $\psi(B)u_t$ is the moving average representation of w_t . Under the alternative hypothesis the model can be written as follows:

$$y_t = \sum_{s=1}^4 \gamma_s D_{s,t} + \varphi(B)[u_t + \sum_{s=1}^4 \delta_s D U_{s,t}] \tag{19}$$

where $\varphi(B)u_t$ is the moving average representation of v_t . The nested auxiliary regression is given as follows:

$$\Delta_4 y_t = \sum_{s=1}^4 \mu_s D_{s,t} + \sum_{s=1}^4 \delta_s D U_{s,t} + \sum_{s=1}^4 \kappa_s D(TB')_{s,t} + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \sum_{j=1}^k c_j \Delta_4 y_{t-j} + \varepsilon_t \tag{20}$$

where ε_t is the error in the approximation. In order to test for the existence of a non-seasonal ($H_0: \pi_1 = 0$) and semi-annual unit root ($H_0: \pi_2 = 0$), the t -values of π_1 and π_2 obtained from the test regression (20) are considered. We can infer that seasonal unit roots would exist if $\pi_3 = \pi_4 = 0$ which can be tested by an F -statistic. The test statistics are denoted by t_1 , t_2 and F_{34} , respectively. It is assumed that the true break date T_B' is unknown. Franses and Vogelsang (1998) show two methods for determining the timing of the break that require regressing equation (20) for every potential break date T_B . The first method chooses the break date which most likely rejects the respective null hypothesis:

$$T_B' = \arg \min_{T_B} t_i(T_B), i=1,2 \text{ and } T_B' = \arg \max_{T_B} F_{34}(T_B) \tag{21}$$

The second method focuses on the coefficients of the break dummy variables and selects the point in time as the break date which maximizes the F -statistic for testing the joint significance of $\delta_s, s = 1, \dots, 4$:

$$T_B' = \arg \max_{T_B} F_\delta(T_B) \tag{22}$$

Harvey et al. (2001) argue that the use of both selection methods in the HEGY seasonal unit root testing procedure leads to tests with considerable spurious rejections in finite samples when a break is found under the null hypothesis. Since the second method tends to choose $T_B' - 4$ as break date under an increasing break size, they modify the second method as in the following way:

$$T_B' = 4 + \arg \max_{T_B} F_\delta(T_B) \tag{23}$$

This method proposed by Harvey et al. (2001) leads to tests with quite stable size for small breaks.

Popp (2007) modifies the seasonal unit root test of Franses and Vogelsang (1998) by representing the data generating process (DGP) as an unobserved component model. Popp considers seasonal unit root test procedure with seasonal mean shifts of unknown timing for quarterly data. Popp argues that the modified test is superior to the procedures used in the literature with respect to size and break date estimation accuracy which is shown by Monte Carlo simulations. The quarterly time series y_t which is consisted of a deterministic component d_t and a stochastic component s_t can be defined as follows:

$$\begin{aligned}
 y_t &= d_t + s_t, \quad s_t = \rho s_{t-4} + \varepsilon_t, \\
 \varepsilon_t &= \Psi^*(L)u_t = A^*(L)^{-1}B(L)u_t, \\
 dt &= \sum_{s=1}^4 \alpha_s D_{s,t} + \Psi^*(L) \sum_{s=1}^4 \theta_s DU'_{s,t} \tag{24}
 \end{aligned}$$

Popp (2007) derives the reduced form of the structural model of the DGP as follows:

$$\begin{aligned}
 y_t &= \rho s_{t-4} + \sum_{s=1}^4 \alpha_s^* D_{s,t} + \sum_{s=1}^4 \zeta_s DU'_{s,t-4} + \sum_{s=1}^4 \theta_s D(TB')_{s,t} \\
 &+ \sum_{j=1}^k \beta_j \Delta_4 y_{t-j} + u_t \tag{25}
 \end{aligned}$$

where $\alpha_s^* = \Psi^*(1)(1-\rho)\alpha_s$ and $\zeta_s = -(1-\rho)\theta_s$. The HEGY auxiliary test regression is given as follows:

$$\begin{aligned}
 \Delta_4 y_t &= \delta_1 y_{1,t-1} + \delta_2 y_{2,t-1} + \delta_3 y_{3,t-1} + \delta_4 y_{3,t-2} \\
 &+ \sum_{s=1}^4 \alpha_s^* D_{s,t} + \sum_{s=1}^4 \zeta_s DU'_{s,t} + \sum_{s=1}^4 \theta_s D(TB')_{s,t} + \sum_{j=1}^k \beta_j \Delta_4 y_{t-j} + e_t \tag{26}
 \end{aligned}$$

Popp (2007) reveals that the most important difference between the test regressions (20) and (26) is that the coefficients indicating the break size appear

in conjunction with the impulse dummy variables $D(T_B')_{s,t}$. Popp then makes a change in the selection method as follows:

$$T_B' = \arg \max_{T_B} F_\theta(T_B) \tag{27}$$

3. Modified HEGY test procedure for testing seasonal unit roots in the presence of seasonal mean shifts in two years

In this section, we extend our analysis by examining the presence of two breaks for seasonal unit roots. For this purpose the HEGY test procedure is modified by adding the structural break dummy variables which become effective only at time TB_1 and TB_2 , where $TB_1 = \lambda_1 T$ with $0 < \lambda_1 < 1$ and $TB_2 = \lambda_2 T$ with $0 < \lambda_2 < 1$ and $1 < TB_1 < TB_2 < T$. The model for quarterly time series under the null hypothesis of one non-seasonal unit root and three seasonal unit roots is written as follows:

$$y_t = y_{t-4} + \varepsilon_t \tag{28}$$

where ε_t is the iid error term. Under the alternative hypothesis, the series y_t does not contain any of these unit roots and can be written as follows:

$$y_t = \sum_{s=1}^4 \mu_s D_{s,t} + u_t \tag{29}$$

where u_t is again iid and D_s are seasonal dummies for the entire sample. The relevant data generation process used in the analyses is given as follows:

$$\Delta_4 y_t = \varepsilon_t \tag{30}$$

where $\varepsilon_t \sim \text{iidN}(0,1)$. The auxiliary regression used in our model is:

$$\varphi(B)y_{4,t} = \mu_t + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \sum_{s=1}^4 \theta_s D(TB_1)_{s,t} + \sum_{s=1}^4 \gamma_s D(TB_2)_{s,t} + \sum_{s=1}^4 \delta_s DU(TB_1)_{s,t} + \sum_{s=1}^4 \lambda_s DU(TB_2)_{s,t} + \varepsilon_t \tag{31}$$

where ε_t is a stationary and invertible ARMA($p+4,q$) process, the $D(TB_1)_{s,t}$ are single observation dummy variables with the following properties:

$$D(TB_1)_{1,t} = 1 \text{ if } t = TB_1 + 1 \text{ (and zero elsewhere)}$$

$$D(TB_1)_{2,t} = 1 \text{ if } t = TB_1 + 2 \text{ (and zero elsewhere)}$$

$$D(TB_1)_{3,t} = 1 \text{ if } t = TB_1 + 3 \text{ (and zero elsewhere)}$$

$$D(TB_1)_{4,t} = 1 \text{ if } t = TB_1 + 4 \text{ (and zero elsewhere)}$$

$$D(TB_2)_{s,t} \text{ can also be considered as single observation dummy variables:}$$

$$\begin{aligned}
 D(TB_2)_{1,t} &= 1 \text{ if } t = TB_2 + 1 \text{ (and zero elsewhere)} \\
 D(TB_2)_{2,t} &= 1 \text{ if } t = TB_2 + 2 \text{ (and zero elsewhere)} \\
 D(TB_2)_{3,t} &= 1 \text{ if } t = TB_2 + 3 \text{ (and zero elsewhere)} \\
 D(TB_2)_{4,t} &= 1 \text{ if } t = TB_2 + 4 \text{ (and zero elsewhere)}
 \end{aligned}$$

where $DU(TB_1)_{s,t}$ are composed to allow for the first structural break under the alternative hypothesis:

$$\begin{aligned}
 DU(TB_1)_{1,t} &= 1 \text{ if } t > TB_1 \text{ and } t \bmod 1 = 1 \text{ (and zero elsewhere)} \\
 DU(TB_1)_{2,t} &= 1 \text{ if } t > TB_1 \text{ and } t \bmod 2 = 2 \text{ (and zero elsewhere)} \\
 DU(TB_1)_{3,t} &= 1 \text{ if } t > TB_1 \text{ and } t \bmod 3 = 3 \text{ (and zero elsewhere)} \\
 DU(TB_1)_{4,t} &= 1 \text{ if } t > TB_1 \text{ and } t \bmod 4 = 4 \text{ (and zero elsewhere)}
 \end{aligned}$$

and $DU(TB_2)_{s,t}$ are composed to allow for the second structural break under the alternative hypothesis:

$$\begin{aligned}
 DU(TB_2)_{1,t} &= 1 \text{ if } t > TB_2 \text{ and } t \bmod 1 = 1 \text{ (and zero elsewhere)} \\
 DU(TB_2)_{2,t} &= 1 \text{ if } t > TB_2 \text{ and } t \bmod 2 = 2 \text{ (and zero elsewhere)} \\
 DU(TB_2)_{3,t} &= 1 \text{ if } t > TB_2 \text{ and } t \bmod 3 = 3 \text{ (and zero elsewhere)} \\
 DU(TB_2)_{4,t} &= 1 \text{ if } t > TB_2 \text{ and } t \bmod 4 = 4 \text{ (and zero elsewhere)}
 \end{aligned}$$

Above $DU(TB_1)_{s,t} = 1 (t > TB_1)D_{s,t}$ and $DU(TB_2)_{s,t} = 1 (t > TB_2)D_{s,t}$ where

$1(\cdot)$ is the indicator function and $t \bmod$ shows the corresponding season of this function. $DU(TB_1)_{s,t}$ can be defined as seasonal dummies that only take non-zero values in the corresponding seasons when $t > TB_1$ and $DU(TB_2)_{s,t}$ can be defined as seasonal dummies that only take non-zero values in the corresponding seasons when $t > TB_2$. The auxiliary regression is augmented by the lagged values of the dependent variable. The lag selection method involves testing for the significance of the coefficient of $\Delta_4 y_{t-k}$ using a 10% significance level two sided t -test which is asymptotically distributed $N(0, 1)$. A maximal value of k , k_{max} , is chosen. The value of k is determined so that the coefficient on $\Delta_4 y_{t-k}$ is significant at the 10% level in a k^{th} order autoregression and the coefficient on the last included lag is insignificant in higher order autoregressions up to the order k_{max} .

4. Design of Monte Carlo simulation

In the Monte Carlo investigation the critical values of the HEGY test in the presence of two structural breaks are generated in a GAUSS programme version 4. The critical values for the small sample distributions are displayed for the following different combinations of deterministic terms in the auxiliary regression as in HEGY (1990) and Franses and Hobijn (1997): 1) no intercept, no seasonal

dummy and no trend, 2) intercept, no seasonal dummy and no trend, 3) intercept, seasonal dummy and no trend, 4) intercept, no seasonal dummy and trend, 5) intercept, seasonal dummy and trend.

These cases refer to different model specifications depending on which deterministic terms are used. The critical values for the one-sided t -test for π_1 , critical values for the t -test for π_2 , the F -test statistics for $\{\pi_3, \pi_4\}$, $\{\pi_2, \pi_3, \pi_4\}$ and $\{\pi_1, \pi_2, \pi_3, \pi_4\}$ are generated. In the Monte Carlo investigation the critical values of the HEGY test presented in Franses and Hobijn (1997) are considered as the basis for comparison with the critical values tabulated in this section. Before generating the critical values in the presence of structural breaks, those considered in Franses and Hobijn are generated for the above 5 particular cases using deterministic terms for 10, 20, 30 and 40 years of quarterly observations. Battal (2007) observes that the critical values are the same for most of the specifications and the sample sizes and are quite similar for some others within the 5% level.

The critical values are sensitive to the position of the break and are represented for a range of known alternative break points which are assumed to be some proportion of λ_1 and λ_2 of the sample size. The critical values was generated for the modified HEGY test procedure for testing seasonal unit roots considering break fractions $\lambda_1 = 0.1, 0.3, 0.5, 0.7, 0.9$ and $\lambda_2 = 0.2, 0.4, 0.6, 0.8$ in a sample size of 72.

5. Application

In this section the modified test procedure for testing seasonal unit roots in the presence of possible shifts in the seasonal means is considered by analysing the first data set that consists of quarterly observations of gross domestic product (GDP), real M1, three months Treasury bill interest rates and the GDP deflator of Turkey for the 1986:1 - 2003:1 period. A second data set of money demand variables also includes the GDP with the base 1987:100, real M2, interest rates on weighted average of 3 and 12 months deposits, expected depreciation and expected inflation. Our data set includes the period in which two financial crises in 1994 and 2001 took place.

5.1 Graphical analyses

A first and natural impression of the properties of the seasonal time series can be obtained by depicting their time series plots. The time series data of the natural logarithms of the quarterly GDP data for Turkey is shown in Figure 1 in the appendix. This graph can also be characterized by the typical patterns of many quarterly macroeconomic time series, i.e. there seems to be an upward trend and there exist explicit seasonal fluctuations. When GDP is always large in the third

quarter due to, e.g. the tourism revenues that the Turkish economy experiences, one can say that GDP data displays a high-seasonal characteristic.

When the series in four quarters and the growth rates of the first differenced variables in Figure 2 and Figure 3 in the appendix are examined, it is observed that the seasonality is not constant as can also be observed in Franses (1994). Marked changes are noticed in the middle and at the end of the sample period. It is interesting to investigate seasonality modelling when the seasonal dummy coefficients are changing over time. This case warrants us to make a discussion about the effect of structural breaks as in Perron (1989, 1990). Therefore, a natural extension of the seasonal unit root testing is the behaviour of the seasonal unit root tests when structural breaks exist.

5.2 Finite sample critical values

We have conducted some Monte Carlo experiments to assess the finite sample properties of the modified seasonal unit root test procedure. The critical values of the HEGY test in the presence of two structural breaks have been generated for the money demand application in the paper. For this purpose, we have initially calculated the random numbers of error terms. The numbers of replications in the Monte Carlo experiments have been set to 40000. Then the structural dummy variables, the deterministic terms such as constant, seasonal dummies, and the trend as well as the dependent variable and the regressors used in Eq. (31) have been generated. The time series data used have been generated by way of employing random number generation for the data generation process given in Eq. (28). Then the ordinary least squares estimation has been implemented and the t_1 , t_2 , t_3 and t_4 statistics which correspond to the t -statistics of the regressors y_1, y_2, y_3 and $y_{3,t-1}$ in Eq. (31) have been calculated, respectively. We apply to a similar estimation procedure for estimating the relevant F -statistics. All in all, 0.01, 0.025, 0.05, 0.10, 0.50, 0.95, 0.975, 0.99 percentiles have been obtained for a sample size 72. λ_1 and λ_2 have been considered 0.5 and 0.85, respectively.

5.3 Original and modified HEGY test results

In this section the original HEGY test using auxiliary regression (1), where the possible presence of seasonal mean shifts is neglected, and the modified HEGY test for testing seasonal unit roots in the presence of possible shifts in the seasonal means in two years have been tried to be estimated. The empirical results of the original HEGY and the modified HEGY seasonal unit root tests are discussed for each variable. While comparing these two HEGY test results it should be mentioned that the original HEGY test can be considered more powerful when a series does not actually have seasonal mean shifts.

The auxiliary regression (31) for testing seasonal unit roots in the presence of breaks is used for hypothesis testing. Since there is *a priori* knowledge of the timing of possible break dates, we calculate λ_1 and λ_2 corresponding to the

exogenous break dates. Assuming that the mean shifts have occurred in the second quarter of 1994 because of the massive economic / financial crisis experienced by the Turkish economy, we set $TB1$ in (31) at 1994: 2 corresponding to $\lambda_1=0.5$. Clearly, seasonality has changed roughly halfway through the sample especially for the GDP series. Closer inspection shows that the change occurs after April 1994 and this is the month in which a major crisis occurred and a well-known stabilization programme started on 5 April 1994. The second major change occurs at 2001: 1 because of another massive economic / financial crisis, therefore we set $\lambda_2=0.85$. By looking at the test results obtained from original HEGY and modified HEGY test procedures given in Table 1 in the appendix, it can be seen that there are clearly some variations in the outcomes. The test results vary based on whether the deterministic terms such as seasonal dummy variables and the trend are included in the model and whether 5% or 1% critical value is used for testing. The critical values are given in Table 2 in the appendix.

The results of the original HEGY test for the GDP show that there are non-seasonal and seasonal unit roots. The results for the GDP series using the modified HEGY test confirm that this variable is $I(1,1)$ having non-seasonal and seasonal unit roots at all frequencies. The empirical results suggest that the results of the original HEGY test for the GDP data are robust to the structural breaks at 1994 and 2001. The results of the original HEGY test for the GDP deflator show that there is non-seasonal unit root at zero frequency and seasonal unit root bi-annual frequency. For the GDP deflator series, when the structural breaks at 1994 and 2001 are allowed in the analysis, the GDP deflator appears to have a seasonal unit root at the bi-annual frequency. The non-seasonal unit root seems to be robust to the deterministic mean shifts. Original HEGY test procedure to seasonal unit roots for real M1 money balances series reveals that there are seasonal unit roots at annual frequencies. The seasonal unit root at the annual frequency which is apparent in the model without breaks disappears when one or more shifts in the seasonal means at 1994 and 2001 are allowed. The non-seasonal unit root seems to be robust to deterministic mean shifts. After taking account of the structural breaks using the modified HEGY test procedure, the real M1 balances are found to be $I(1)$. Original HEGY test procedure for the Treasury bill interest rates estimates that there is a non-seasonal unit root at zero frequency only. The results of modified HEGY test for the Treasury bill interest rates confirm the results of original HEGY test procedure. The non-seasonal unit root is found to be robust to the deterministic mean shifts at 1994 and 2001. With no seasonal mean shift, the results of the original HEGY test procedure for the real M2 series show that there is a non-seasonal unit root at the zero frequency. For the real M2 money balances series, when the structural breaks at 1994 and 2001 are allowed in the analysis, the real M2 money balances appear to have a seasonal unit root at the annual frequency. The non-seasonal unit root seems to be robust to deterministic mean shifts. Original HEGY test results for

the expected depreciation series show that this variable is stationary. However, when the breaks are allowed by means of the modified HEGY test procedure, expected depreciation appears to be integrated of order 1 with non-seasonal unit root at zero frequency. According to the modified HEGY test results, expected inflation appears to have seasonal unit roots at annual frequencies. Finally, original HEGY test procedure for interest rates on deposits suggests that there exists a non-seasonal unit root at zero frequency only. The results of the modified HEGY test procedure for interest rates on deposits confirm the results of original HEGY test procedure.

The original HEGY test results for the GDP, the Treasury bill interest rates and interest rates on deposits are found to be robust to the seasonal mean shifts in one or more seasons at 1994 and 2001. This finding is expected for interest rate variables as they are unlikely to exhibit seasonal variation. Thus we do not expect a change in the outcome of the original and modified HEGY test results for interest rates. The original HEGY test results for the GDP deflator, real M2 and expected inflation series mean that these variables are $I(1)$. When the structural breaks are allowed at 1994 and 2001 for the GDP deflator, a seasonal unit root at the bi-annual frequency appears. For real M2 and expected inflation series seasonal unit roots appear at annual frequency. Therefore, GDP deflator, real M2 and expected inflation become seasonally $I(1,1)$ once the seasonal mean shifts are allowed. As price series show a typical seasonal pattern in Turkey, it is expected to observe seasonal unit roots for expected inflation. All in all, we can conclude that in general the modified HEGY tests produce mixed results about the integration of the variables when compared with the results of the original HEGY test procedure.

Concluding remarks

Due to the changing nature of the seasonal patterns in economic time series, there has been an increased interest for incorporating these changes into the seasonal unit root testing. In this paper, we propose a modification of the Hylleberg et al. (1990) (HEGY) procedure based on innovative outlier model for testing seasonal unit roots. Our modified model can be considered as an extension of Franses and Volgelsang (1995) in the case of known break dates.

We consider two major economic / financial crises, namely 1994 and 2001 crises, in the Turkish economy as exogenous break points. These crises have institutional changes in the economy and are accepted as major changes in many studies on the Turkish economy such as Selcuk (1994), Akcay et al. (1997), Selcuk and Ertugrul (2001) and Civcir (2003). The modified HEGY seasonal unit root test procedure is applied to the GDP, GDP deflator, real M1, real M2, the Treasury bill interest rates, expected depreciation, expected inflation and

the interest rates on deposits in the Turkish economy. For the Treasury bill interest rates, real M1, expected depreciation and interest rates on deposits series, no seasonal unit roots seem to be observed. Only the unit root at zero frequency is found. Therefore, one can assume for these variables an approximate deterministic seasonality model as put forward in Miron (1996) such that the seasonal dummy parameters reflect the seasonal cycle. For the GDP deflator, real M2 and expected inflation data, seasonal unit roots appear in the sense that leads us to infer that a seasonal unit root test is likely to be appropriate for these variables. As the GDP variable is found to have non-seasonal and seasonal unit roots according to both the original and modified HEGY test procedures, seasonal unit root procedure is relevant for the GDP data.

Demand for money plays a major role in assessing the appropriateness of the discretionary policies carried out by the policy makers. Many macroeconomic time series data including the determinants of money demand exhibit strong seasonality but until recently only a few papers have examined the detailed properties of the seasonal patterns. Central bankers must consider whether there exist a stochastic or a deterministic seasonality for the variables determining money demand relationship in Turkey. In this line, our *ex-post* findings reveal that the gross domestic product and inflation variables display significant seasonal characteristics which result in important consequences in the conduct of the monetary policy. If the seasonality in such variables has been ignored, the course of aggregate demand and also both the long- and the short-run characteristics of the money demand relationships would not possibly be able to accurately forecasted in the subsequent periods of the economy.

Based on whether or not the structural breaks are considered, we find that some differences can be taken place within the estimation results obtained in our empirical modelling. Thus future papers must consider these issues of interest in a more elaborately way to confirm the basic results of this paper and must also analytically extend the HEGY seasonal unit root procedure for the multi structural break cases where the effects of more than two structural breaks on the seasonal unit root tests are taken into account.

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APPENDIX

Figure 1. Gross Domestic Product

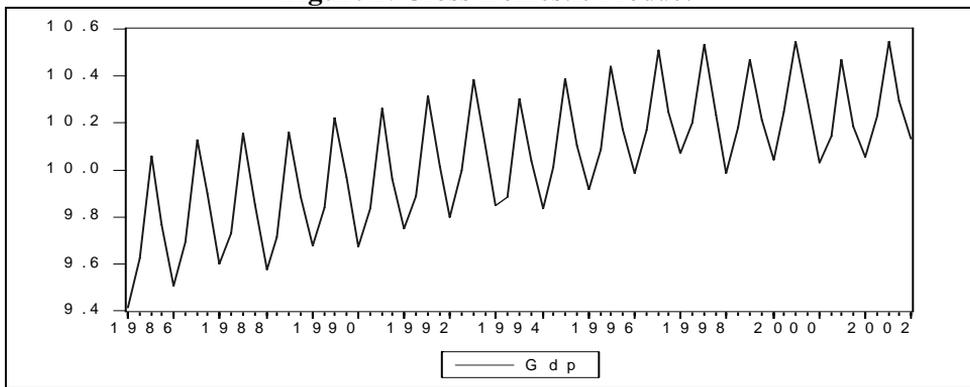


Figure 2. GDP Observed in Quarters 1, 2, 3 and 4

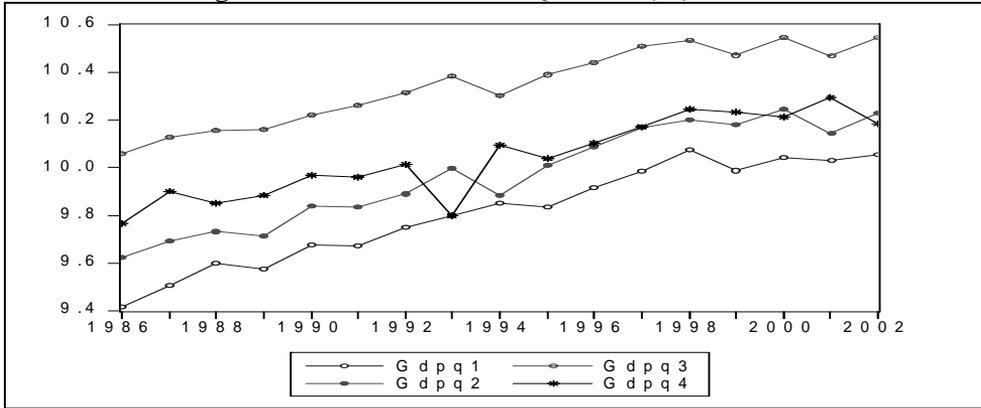


Figure 3. The First Differences of GDP in Quarters 2, 3 and 4

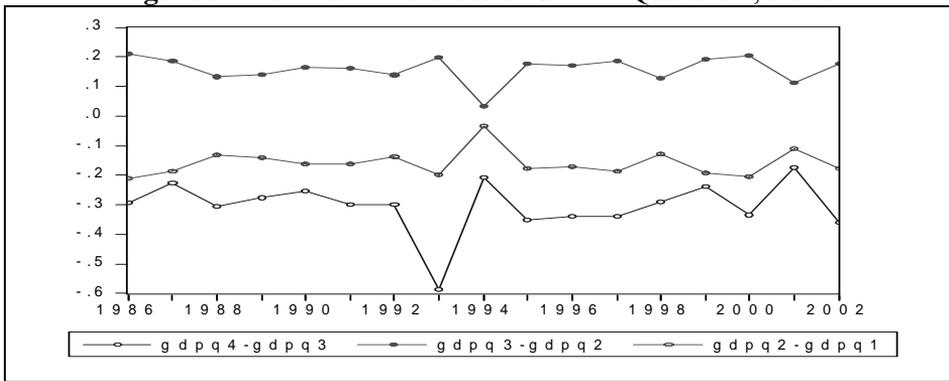


Table 1. Comparison of Original HEGY and Modified HEGY Test Results

Variables	Original HEGY	Modified HEGY	Changes
GDP	Unit roots at π_1, π_2, π_3 and π_4 Transformation: $(1-B)(1+B)(1+B^2)$	Unit roots at π_1, π_2, π_3 and π_4 Transformation: $(1-B)(1+B)(1+B^2)$	No modification
GDP Deflator	Unit root at π_1 Transformation: $(1-B)$	Unit roots at π_1 and π_2 Transformation: $(1-B)(1+B)$	Unit root at π_2 appears

Real M1	Unit roots at π_3 and π_4 Transformation:(1+B ²)	Unit root at π_1 Transformation:(1-B)	Unit roots at π_3 and π_4 disappears, unit root at π_1 appears
Int. Rates	Unit root at π_1 Transformation:(1-B)	Unit root at π_1 Transformation:(1-B)	No modification
Real M2	Unit root at π_1 Transformation:(1-B)	Unit roots at π_1, π_3 and π_4 Transformation:(1-B)(1+B ²)	Unit roots at π_3 and π_4 appears
Expected De-preciation	No unit root, Stationary Series	Unit root at π_1 Transformation:(1-B)	Unit roots at π_1 appears
Expected Inflation	Unit root at π_1 Transformation:(1-B)	Unit roots at π_1, π_3 and π_4 Transformation:(1-B)(1+B ²)	Unit roots at π_3 and π_4 appears
Dep.Int. Rates	Unit root at π_1 Transformation:(1-B)	Unit root at π_1 Transformation:(1-B)	No modification

Table 2. Critical Values for the Turkish Data Set when there are Two Breaks at 1994 and 2001, T=72
 $\lambda_1=0.5$ $\lambda_2=0.85$ no constant, no seas dummy, no trend (1)

	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
t1	-3.28	-2.89	-2.57	-2.22	-0.96	0.45	0.86	1.20	1.61
t3	-3.57	-3.21	-2.87	-2.47	-1.05	0.38	0.80	1.14	1.56
t4	-2.33	-1.94	-1.62	-1.25	-0.01	1.24	1.61	1.92	2.29
t2	-3.29	-2.90	-2.59	-2.24	-0.97	0.45	0.86	1.20	1.60
F34	0.02	0.04	0.08	0.18	1.19	3.89	5.02	6.16	7.63
F1...4	0.15	0.25	0.35	0.52	1.58	3.55	4.32	5.07	6.08
F 2...4	0.07	0.14	0.22	0.37	1.45	3.71	4.58	5.49	6.60

T=72 $\lambda_1=0.5$ $\lambda_2=0.85$ constant, no seas dummy, no trend (2)

	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
t1	-4.01	-3.65	-3.36	-3.03	-1.94	-0.80	-0.45	-0.14	0.23
t3	-3.60	-3.22	-2.88	-2.47	-1.06	0.36	0.77	1.11	1.53
t4	-2.28	-1.90	-1.58	-1.22	0.00	1.24	1.60	1.91	2.28
t2	-3.28	-2.90	-2.59	-2.24	-0.98	0.43	0.84	1.18	1.57
F34	0.02	0.04	0.08	0.18	1.18	3.89	5.01	6.14	7.64
F1...4	0.31	0.46	0.63	0.89	2.23	4.53	5.40	6.24	7.35
F2...4	0.07	0.14	0.22	0.37	1.45	3.71	4.58	5.46	6.65

T=72 lambda1=0.5 lambda2=0.85 constant, no seas dummy, trend (3)

	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
t1	-4.39	-4.04	-3.74	-3.40	-2.30	-1.15	-0.80	-0.49	-0.12
t3	-3.62	-3.22	-2.88	-2.47	-1.06	0.35	0.75	1.08	1.50
t4	-2.23	-1.86	-1.55	-1.20	0.01	1.22	1.57	1.88	2.26
t2	-3.27	-2.89	-2.58	-2.23	-0.98	0.41	0.81	1.14	1.53
F34	0.02	0.04	0.08	0.17	1.17	3.87	4.99	6.11	7.58
F1...4	0.40	0.60	0.81	1.11	2.62	5.08	5.98	6.91	8.06
F2...4	0.07	0.14	0.22	0.37	1.43	3.68	4.55	5.43	6.61

T=72 lambda1=0.5 lambda2=0.85 constant, seas dummy, no trend (4)

	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
t1	-3.99	-3.65	-3.34	-3.01	-1.92	-0.80	-0.47	-0.16	0.22
t3	-4.82	-4.42	-4.10	-3.75	-2.56	-1.36	-1.03	-0.73	-0.37
t4	-2.56	-2.12	-1.77	-1.37	-0.01	1.36	1.75	2.09	2.49
t2	-4.01	-3.65	-3.35	-3.01	-1.93	-0.81	-0.47	-0.17	0.19
F34	0.31	0.61	0.93	1.41	4.01	8.09	9.57	11.06	12.97
F1...4	1.02	1.38	1.71	2.16	4.34	7.64	8.85	10.01	11.62
F2...4	0.71	1.05	1.39	1.86	4.22	7.78	9.05	10.29	11.99

T=72 lambda1=0.5 lambda2=0.85 constant, seas dummy, trend (5)

	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
t1	-4.39	-4.02	-3.72	-3.39	-2.28	-1.15	-0.80	-0.49	-0.13
t3	-4.81	-4.41	-4.09	-3.74	-2.55	-1.36	-1.02	-0.72	-0.38
t4	-2.48	-2.05	-1.70	-1.31	0.04	1.37	1.76	2.10	2.51
t2	-4.00	-3.64	-3.33	-3.00	-1.93	-0.81	-0.47	-0.17	0.16
F34	0.30	0.60	0.92	1.40	3.98	8.02	9.50	11.00	12.88
F1...4	1.16	1.52	1.89	2.39	4.72	8.20	9.47	10.72	12.31
F2...4	0.69	1.03	1.39	1.85	4.18	7.72	8.99	10.23	11.84