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How Do Mean Division Shares Affect Growth and Development?

Summary: The Gini coefficient is widely used in academia to discuss how income inequality affects development and growth. However, different Lorenz curves may provide different development and growth outcomes while still leading to the same Gini coefficient. This paper studies the development effects of “mean division shares”, i.e., the share of income (mean income share) held by people whose household disposable income per capita is below the mean income and the share of the population (mean population share) with this income, using panel data. Our analysis explores how this income share and population share impact development and growth. It shows that the income and population shares affect growth in significantly different ways and that an analysis of these metrics provides substantial value compared to that of the Gini coefficient.

Key words: Income inequality, Mean division shares, Growth.

JEL: C36, D31, O11.

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Academia has continued to debate how income distribution affects development and growth since Kuznets’s hypothesis was proposed in 1955. There are two fundamental issues with this issue. The first relates to measuring (relative) income inequality. Summary indices are the most popular tools for measuring inequality; economists have found that many summary measurements are unable to provide strictly Lorenz ranking regarding income distributions, and that the correlations between growth and income inequality may differ in inequality measurement. The other question relates to the mechanics of how income inequality plays a role in production, i.e., how income inequality should enter a production function. Specifically, the functional specification of the macroeconomic effects of income inequality requires justification. This paper seeks to address with these two issues.

This paper studies the development effects of “mean division shares”, i.e., the share of income (mean income share) held by people whose household disposable income *per capita* is below the mean income and the share of the population (mean population share) with this income, using panel data. Our analysis explores how these income share and population shares impact development and growth.

The paper consists of five sections. Section 1 summarizes relevant studies; Section 2 is about the model specification; Section 3 is about the data we will use and the estimation for our measurements; Section 4 is the empirical analysis and Section 5 concludes.

1. Literature Review

In new classic growth theory, a concave production function ensures that perfect competitive economies will converge on a steady state, suggesting that income inequality would eventually not matter for growth and development. The real world does not reflect perfect competition and the so-called "steady state", and economists have been presenting theoretical and empirical findings on the correlations between inequality and growth that unfortunately have not been in line with each other.

Using within-fixed effects regressions, Torsten Persson and Guido Tabellini (1994) and Kristin J. Forbes (2000) find a negative relationship between changes in inequality and changes in economic growth rates. Hengyi Li and Henfu Zou (1998) find a positive relationship between income inequality and growth. Dierk Herzer and Sebastian Vollmer (2012) use heterogeneous panel co-integration techniques, finding that inequality has a negative long-term effect on income.

Robert J. Barro (2000) uses three-stage least squares regressions on a panel of countries and finds no overall relationship between inequality and growth, but he claims that there is a negative relationship in the subpanel of poor countries and a positive relationship in the subpanel of rich countries.

Abhijit V. Banerjee and Esther Duflo (2003) introduces the quadratic form of lagged changes of the Gini index and uses random-effects models to show that the growth is an inverted U-shape curve of net changes in the Gini index; changes in inequality in any direction are associated with reduced growth in the next period.

These studies ignore interactive terms between income inequality and productive factors. The cross items are not described by country fixed-effects or random-effects, but they are correlated with both inequality and productive factors. Thus, the missing cross-effect variables resulted in previous research being biased and inconsistent.

We also find strong serial correlations in the panel data that indicate that both fixed-effects and random-effects models are inappropriate. We employ robust system GMM estimators with a dynamic panel model to address heteroskedascity and endogeneity, which gives us consistent estimates. The maximum number of instruments is chosen so that the validity of over-identification is not rejected and the null hypothesis of no autocorrelation in the first-differenced errors is also not rejected.

Sarah Voitchovsky (2005) finds that the profile of income distribution matters for economic growth. This idea is supported in this paper using the mean division shares, the share of income held by people whose *per capita* household disposable income is below the mean income (mean income share) and the share of population with this income (mean population share).

Robert Duval Hernández, Gary S. Fields, and George H. Jakubson (2014) find that it is theoretically possible to have rising or falling inequality along with convergent or divergent mobility (changes in income) in times of both economic growth and decline. This finding explicitly states that the correlation between inequality and growth is nonlinear and non-monotonic, and this paper presents strong empirical evidence in support of that proposition.

We present three primary contributions in this paper. First, we use a different measurement of income inequality to explore its macroeconomic effects. We describe income inequality by the mean population share and mean income share held by the people whose household disposable incomes *per capita* are not more than the national mean, which enables us to see how population and income shares affect development and growth.

Second, we allow inequality to interact with production factors, which controls for the effects of different structures and institutions in an economy. We run a Wald test for the significance of cross items and it shows a very strong joint significance for these items.

Last, we find that the profile of income distribution matters in determining the macroeconomic effects of income inequality. The mean income share may have a positive effect on GDP *per capita*, but mean population share may not. Meanwhile, we find significant effects of the changes in mean division shares on GDP *per capita* in the next period. The development and growth form an inverted U-shape function of the changes in mean population share. This is similar to the findings of Banerjee and Duflo (2003), who use the Gini index to measure income inequality. However, changes in mean income share show the opposite relationship with development and growth.

2. The Model

2.1 Function Specification

We use the Cobb-Douglas production function to explain the interactions between inequality and production input factors and the role income inequality plays in production. Assume there are only two productive factors in an economy, *per capita* labor l and *per capita* capital stock k. y denotes GDP *per capita*, α is labor income share, and the production function is $y = l^{\alpha}k^{1-\alpha}$. The production function in log form is:

$$lny = \alpha lnl - \alpha lnk + lnk. \quad (1)$$

Labor share α can be considered a measurement of income inequality because it denotes the income share of the employed people who rent out their labor. We may let l denote the population share whose people hold income share α, and the pair values (l, α) describe the income inequality in this model economy. At full employment, this population share l would be unit, which is different from our mean population share, which is to be defined in Subsection 3.1. The pair values (l, α) do not tell us how many people are relatively poor and how poor they are, which will be resolved by our mean division point.

Therefore, Equation (1) shows that the log equation of output for a Cobb-Douglas technology economy consists of the interactions between inequality, α, and input factors, lnl and lnk; and a measurement of income inequality α exists in the production function.

Now we turn to an empirical model to explore the macro-effects of income inequality. For that study, we may want to combine data from both developed and developing economies to one regression function. This approach would allow a one-period lag in GDP to take quadratic form, enabling a test of the nonlinearity specification to refine the model. This implies that an economy's development level is denoted by the input factors and the one-period lag in GDP *per capita*.

To be concise, we submit the letter of log for all variables. Let GDP_t denote the log form of *per capita* output at period *t*; $Input_{jt}$, the log form of input factor j at period *t*; α_j, β_j, and γ_j are the parameters to be estimated; mds_t, the mean division shares (mean population and income shares to be defined in the Subsection 3.1) at period *t*; $f(..)$ is a quadratic function of mds_t and mds_{t-1}; and ε_t the error term. The variable mds_t denotes mean division share (MDS) at *t*, which is either mean population share or mean income share to be defined in the Subsection 3.1.

We apply the following dynamic level output Equation (2), which is generated from Equation (1) at period t and $t-1$, to a panel data, in which the panel subscript is omitted:

$$GDP_t = \alpha_0 + \alpha_1 GDP_{t-1} + \alpha_2 {GDP_{t-1}}^2 + \alpha_3 f(mds_t, mds_{t-1}, \Delta mds_t, \Delta mds_{t-1}) + \sum_{p=0,1} \sum_j \left[\beta_{t-p,j} Input_{jt-p} mds_{t-p} + \gamma_j Input_{jt-p}\right] + \varepsilon_t. \tag{2}$$

We do not consider technology growth specifically in the model. Changes in MDS (mean division share), working hours, and capital formation on the right hand side of (2) partly express the effects of technology growth because this technology growth is generally considered exogenous and it can affect income distribution, investment and employment as well; meanwhile we use 5-year average data and GMM instruments on the first difference of the model so that technology shocks will be partly smoothened out. Income inequality is expressed by MDS in this paper. It is assumed in (2) that MDS and its interactions with other inputs may affect development (GDP) and growth (changes in GDP), and we use a *t*-test and an *F*-test to verify this assumption; later empirical results show that we cannot reject the assumption. Note that GDP and its one-period lag in this model allow us to explain both the development (level of GDP) and growth (changes in GDP) effects of MDS at the same time.

Here we include only a one-period lag for GDP on the right-hand side because we have included one-period lags for the changes in MDS and other inputs, which implies that GDP lags in period two and later are implicitly described. In fact, when both lag period one and lag period two GDP are included, the estimate of lag period two is individually insignificant and the estimates for all MDS items are significantly affected; meanwhile, the observations decrease to 138 from 171, a 19% decrease in observations. We also found similar results when period two lags in other input factors were included. Therefore, we only include the one-period lag in GDP and input factors in the model.

To address the endogeneity issue of (2), we use all one-period and later lags of the changes in all endogenous and predetermined explanatory variables to instrument the first difference equation, a one-period lag of all explanatory variables to instrument the level equation, and all exogenous explanatory variables as standard instruments. This is the one-step system GMM estimator created by Manuel Arellano and

Stephen Bond (1991). We run the estimation on STATA and choose the robust option. We choose the maximum number of instruments so that the validity of over-identification and zero correlation in the first-differenced errors are not rejected in the Sargan test and Arellano-Bond test, respectively. This estimation is consistent, but it is sensitive to the number of instruments, which might indicate bias.

2.2 Test for Nonlinearity Specification

Banerjee and Duflo (2003) add quadratic forms of the level Gini index and one-period lags of changes in the index to explain growth, but they neither consider interactions between inequality and factor inputs nor test for nonlinearity specification. We address these issues in this paper.

We apply RESET (Jeffrey M. Wooldridge 2009, p. 303) to test the nonlinearity specification of Equation (2) in the following way. First, we obtain the fitted value, $\widehat{GDP}$, from (2) and generate the fitted values $\widehat{GDP}^2$and $\widehat{GDP}^3$; next, we set $\widehat{GDP}^2$ and $\widehat{GDP}^3$ as explanatory variables to obtain (3) bellow:

$$\begin{aligned} GDP_t = \alpha_0 + \alpha_1 GDP_{t-1} + \alpha_2 {GDP_{t-1}}^2 + \alpha_3 f(mds_t, mds_{t-1}, \ \Delta mds_t, \Delta mds_{t-1}) \\ + \sum_{p=0,1} \sum_j \left[\beta_{t-p,j} Input_{jt-p} mds_{t-p} + \gamma_j Input_{jt-p}\right] \\ + \widehat{GDP}^3 + \widehat{GDP}^2 + \varepsilon_t. \end{aligned} \tag{3}$$

Third, we run (3) in the same way as (2) and perform a Wald test for the joint significance of $\widehat{GDP}^2$and $\widehat{GDP}^3$. If they are jointly significant, then we reject the nonlinearity specification of (2); otherwise, we do not reject the specification.

2.3 Function Choice

In Equation (2), mean division shares and productive factors enter with current and one-period lags. To refine the model, we use a Wald test (RESET) for the joint significance of the current and one-period lags for each variable, we keep the current and one-period lags of a variable if they are jointly significant, and we also use RESET to test for the joint significance of all variables for which their current and one-period lags are jointly insignificant. A group of individually insignificant variables are kept in the model when they are jointly significant and their presence affects other variables' significance; otherwise, they are dropped from the model.

There can be multiple combinations of mds_t and mds_{t-1} for the function $f(mds_t, mds_{t-1})$. For instance, we may also test for the specification in Banerjee and Duflo (2003); lagged changes in inequality are entered in $f(mds_t, mds_{t-1}, \Delta mds_{t-1}, \Delta mds_{t-1})$. To refine this function, we use RESET again to test for the joint significance of each possible combination of $mds_t, mds_{t-1}, \Delta mds_{t-1}$, and Δmds_{t-1}. An insignificant variable is kept only when its exclusion will dramatically affect the significance of other variables. There can be multiple versions of (2) due to changes in the function form of *f*(...), some of which may not be rejected by the above nonlinearity specification test, in which case we need to further rank these functions according to their explanatory power.

We apply the Davidson-Mackinnon test and take the following steps to choose the more powerful one of two optional functions that are not rejected by the nonlinearity specification test. Let $\widehat{GDP}_1$ and $\widehat{GDP}_2$ be the fitted values for the two optional functions. We put the fitted value of one optional function into the other and run the new regressions with the same method used for the optional functions. If only one of the two fitted values is significant, then we choose the function with that fitted value. If both fitted values are either significant or insignificant, then we cannot determine which is more powerful than the other, and we keep both options.

The quadratic function of Δmds_{t-1} is found to be jointly significant, and the quadratic item is individually significant as well regardless of the function form of level mean division shares. Thus, a change in mean division shares does affect development and growth in the next period. In fact, the regressions show that development and growth form an inverted U-shape function of the changes in mean population share but a U-shape function of the changes in mean income share. That is, the changes in mean income share and mean population share present opposite effects; thus, the profile of income distribution matters for its macroeconomic effects.

3. Data

Data quality is an important issue for empirical studies. Many data sets on income distribution have been collected for many countries by different agents using different statistical units, survey methods and income definitions. It is common to find different measures for a country's income inequality in different data sets due to changes in statistical methods, definitions of income and sample errors.

Fortunately, the income distribution projects at WIDER have been working on this issue. Project staff have professionally examined all previous income distribution data sets, published all data details, ranked their quality, and made their work publicly available; their latest release is the WIID3b. This paper uses the WIID3b data to estimate income inequality from previous data sets using *per capita* household disposable income. We retrieved only high- and average-quality WIID3b data that had quintile or decile points for income distribution.

3.1 Mean Division Point of Income Distribution

First, we mathematically define mean division point and then discuss their estimation using different data sources.

Definition. The mean division point (MDP) of a smooth Lorenz curve is located at the point with unit slope.

The corresponding coordinates are called mean population share and mean income share. Mean population share (MPS) and mean income share (MIS) are called mean division shares, hereafter MDS. MDP is not actually a new concept in the literature, but it has not been studied as a measurement of income inequality. The Pietra ratio (Robin Hood index) is the difference between MPS and MIS. Liang Frank Shao (2011) shows that the Gini index is approximately 1.3 times the Pietra ratio using the simple OLS estimator and the data version WIID2b. I have revised this estimation recently using fixed-effects panel estimator and the latest data version WIID3c, and noted that the Gini index could be the difference between MPS and 0.74 times of MIS.

Let $f(w)$ be the probability density function of income distribution, with *w* denoting the income level. Accordingly, $F(w)$ is the cumulative probability function of population share with individual income no more than *w*, μ is the mean *per capita* income in the economy, and (x, y) is a point on the economy's Lorenz curve. Then, we have the following Lorenz curve:

$$y(x) = \left(\frac{1}{\mu}\right)\int_0^{F^{-1}(x)} w dF(w) = \left(\frac{1}{\mu}\right)\int_0^{x} F^{-1}(t)dt. \tag{4}$$

The MDP(x^*, y^*) on the Lorenz curve is defined by the following equations:

$$\begin{cases} x^* = F(\mu) \\ y^* = \left(\frac{1}{\mu}\right)\int_0^{F(\mu)} F^{-1}(t)dt\,. \end{cases} \tag{5}$$

The MDP consists of people whose *per capita* household disposable incomes are not more than the national mean. This point is unique for strictly smooth Lorenz curves, but economies with different mean incomes may have the same MDP. Thus, we must denote the corresponding development level when we compare MDP; it is more straightforward to compare MDP for countries with similar *per capita* GDP.

3.2 Data Sources

Our variables include GDP, population, capital stock, human capital, employment, investment, net export, and average working hours, labor share, government consumption, and real inflation rate. These were retrieved from the PWT8.0 (University of Groningen's Penn World Table 2014)[1].

The data for income distribution are taken from the WIID3b (World Income Inequality Database) of the United Nations University World Institute Development Economics Research - UNU-WIDER (2015)[2]. The WIID3b is a panel database built from different earlier works on the income distribution of countries around the world. These data were collected by different agents and vary with respect to the definitions of income, coverage of the sample area, household, age, and population. We choose the data and estimate the MDS with the following criteria to accommodate the variety of different data sources in our regression model.

Income is defined as disposable income (refer to the "World Income Inequality Database User Guide", Table 1 on page 6, and the definition on page 10) measured by household *per capita*. Sample coverage is nationwide and covers all ages of the adult population. However, if the data available were not sampled by these standards, we either transfer them by the method discussed in Subsection 3.3 or take an approx-

[1] **University of Groningen.** 2014. Penn World Table - The Database. http://www.rug.nl/ggdc/productivity/pwt/pwt-releases/pwt8.0 (accessed April 02, 2014).

[2] **United Nations University World Institute Development Economics Research (UNU-WIDER).** 2015. World Income Inequality Database. https://www.wider.unu.edu/download/wiid-v30a (accessed April 14, 2015).

imation for a few observations. For instance, disposable income is approximated by the squared root of economic family equivalence for Canada.

The WIID3b ranks the quality of an observation according to its income definition, survey quality, and sample methods, and we use only high- and average-quality data. Please refer to the WIID3b user guide for the quality definition.

The data-set includes some observations for which the surveyed income was either gross (monetary) income or disposable monetary income or for which income was measured by household or household equivalence (OECD method, HBAI, square root; please see the WIID3b user guide for these definitions), which can be used to impute the Gini index that agrees with our standards. This enables us to enlarge our dataset as much as possible.

An observation of income distribution will be kept in our dataset even if it does not satisfy our standards, but the Gini index that satisfies our standards is available for the year of the observation. In these cases, the Gini index can be used to estimate the MDS and we discuss how to estimate the MDS to satisfy our statistical standards in Subsection 3.3.

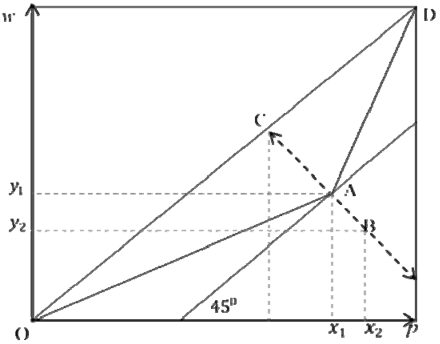
One country's data may come from different sources to enlarge the dataset, and an observation from one source may differ from those of other sources for the same year, even if the same statistical method was used. When this occurs, we choose the data with more observations and/or the latest version. However, we have to accept that statistical errors exist in the pool of different data sources that cannot be completely overcome.

Our analysis uses unbalanced panel data from the PW8.0 and WIID3b from 1950 to 2011 from 70 countries. The data used are capable of being cleaned using the above standards and include the variables that are required in our models. All variables are measured by household *per capita* using the log value, and variables are demeaned with the panel mean if they take quadratic form or are used in interactive items; demeaned variables are used to reduce perfect collinearity. The changes in mean division shares that are to be applied to the model are the 5-year averaged changes in mean division shares, not the changes in two consecutive 5-year averaged changes in mean division shares. In fact, it is only the average changes in mean division shares, not the changes in average mean division shares that result in significant effects on development and growth.

3.3 Estimate MDS with a Given Gini Index

Many datasets that were not built with the methodology we use in this paper. Therefore, it is important to find a method of estimating MDS so that merged data from various sources will satisfy the sole statistical unit and concept definition adopted in this paper. The Lorenz curve changes with different income and population units as well as with different definitions of income. In these cases, the new Lorenz curve is unknown, but if the Gini indices from before and after the change are known, then the MDS of the new Lorenz curve can be approximated using the following method.

To simplify the problem, we consider only changes that make the Lorenz curve shift along the orthogonal direction of the tangent line at the MDP, so that the corresponding Gini index always changes. We call this assumption as an orthogonal shift. This simplification assumes that the change in measuring unit or income definition proportionally affects each point on the Lorenz curve. Figure 1 below shows the shifting of a triangle Lorenz curve. Let g_1 be the Gini index of the Lorenz curve, let $\widehat{OAD}$ and $A(x_1, y_1)$ be its MDP; g_2 is the Gini index and $B(x_2, y_2)$ is the MDP of the Lorenz curve $\widehat{OBD}$ after a change in the measuring unit or income definition.



Source: Own elaboration.

Figure 1 Orthogonal Shifting of the Lorenz Curve

The assumption of a shifting change is not sufficient to describe point B by point A, but the problem can be resolved if we further approximate the ratio g of Gini indices by the squared ratio of heights on the common bottom line $\overline{OD}$ of the two triangles, ΔOAD and ΔOBD. That is, if the two triangles ΔOAD and ΔBDD are assumed to be similar when the shift is very small, then:

$$\frac{g_2}{g_1} \approx \left(\frac{BC}{AC}\right)^2 = g. \tag{6}$$

Employing Assumption (6), we have the following results:

$$\sqrt{\frac{g_1}{g_2}} \approx \frac{AC}{BC} = \left(x_1 - \frac{x_1 + y_1}{2}\right) \Big/ \left(x_2 - \frac{x_1 + y_1}{2}\right) = \left(\frac{x_1 + y_1}{2} - y_1\right) \Big/ \left(\frac{x_1 + y_1}{2} - y_2\right)$$
$$\therefore \begin{cases} x_2 \approx .5x_1\left(1 + \sqrt{g}\right) + .5y_1\left(1 - \sqrt{g}\right) \\ y_2 \approx .5x_1\left(1 - \sqrt{g}\right) + .5y_1\left(1 + \sqrt{g}\right) \end{cases}. \tag{7}$$

Using the Gini ratio g and MDS (x_1, y_1) before the shift in the Lorenz curve, we can estimate the new MDS(x_2, y_2) after the shift with Equations (7). Fortunately, the Gini index is widely available with different definitions and statistical methods in the WIID3b dataset, so that the Gini ratio g can be easily computed. Note that if the Gini indices are the same due to the statistical changes in the data, then we will not be able to estimate the MDS we need, and we will likely have to drop the observation when this occurs. Fortunately, there is not such case in the data we use, which may imply that our orthogonal shift assumption is proper for the dataset.

We can obtain the Gini ratio when the Gini indices are available before and after a change in the statistical method. Otherwise, we choose the average Gini ratio for the previous two or three periods when they satisfy our standards.

Finally, we enlarge our dataset having used all current income distribution resources and maintain a uniform statistical unit within each country, but we are still unable to apply the sole statistical unit to all countries due to lack of information. For instance, income was defined using disposable monetary income for the Republic of Korea, Belgium, Switzerland, and Australia; and gross monetary income for New Zealand and Argentina; and gross income for Honduras, Ukraine, and Uruguay. The statistical unit of income was the square rooted household equivalence for Republic of Korea and Norway; the square rooted economic family equivalence for Canada; the OECD method household equivalence for Australia, Austria, France, Greece, Ireland, the Netherlands, and Portugal; household equivalence (HBAI) for the United Kingdom; and tax unit *per capita* for Switzerland.

4. Empirical Analysis

4.1 Estimated MDS

The estimated annual MDS and original MDS are summarized in Table 1 below; the table shows that the overall differences are not large regarding the means and standard deviations. Figure A1 and Figure A2 in the Appendix show the differences, which are large for some observations, for the Gini coefficient and MDS, respectively.

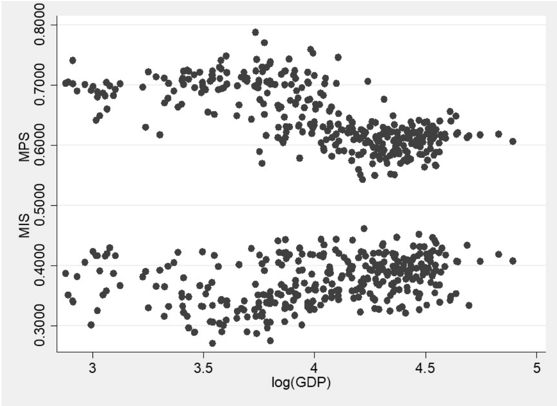
Table 1 Summary of the Estimated and Original Annual MDS

Variable	Obs	Mean	Std.	Min	Max
Original Gini	1128	.3652	.1008	.1970	.6600
Estimated Gini	1124	.3701	.1047	.2070	.6726
Original MPS	1126	.6410	.0495	.5353	.8075
Estimated MPS	1122	.6417	.0506	.5335	.8075
Original MIS	1126	.3782	.0424	.2510	.4696
Estimated MIS	1122	.3774	.0409	.2510	.4696

Source: Own elaboration.

Table A1 in the Appendix contains a data summary for the 5-year average variables. From 1124 annual observations of MDS and annual observations of other variables though 1996, we are able to calculate 329 non-overlapping five-year average observations for all level variables and 292 observations for the average of annual changes in the MDS.

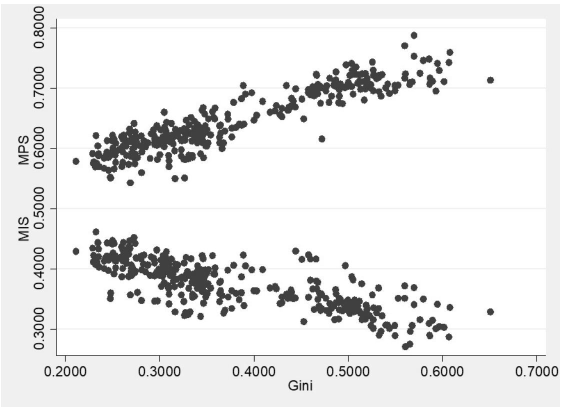
Some audiences may be interested in how MDS changes over time or in GDP. We draw MDS with GDP *per capita* in Figure 2 below, which shows that using the 5-year average data, MPS rises slightly (in, say, the developing stage) and then decreases greatly later (the developed stage), and MIS stays stable first and then increases slightly in GDP *per capita*. Thus we can see that the Pietra ratio increases slightly at first and then decreases greatly later in the GDP in the panel data.



Source: Own elaboration.

Figure 2 5-Year Average MDS against GDP per capita

We also find in the data that the Gini coefficient is positively correlated with MPS and negatively correlated with MIS, which suggests that MDS may be able to describe different information from the Gini coefficient. Figure 3 below shows the observations of MDS and the Gini index in the panel data.



Source: Own elaboration.

Figure 3 Observations of MDS and the Gini Coefficient

4.2 Regression Results

As mentioned in Subsection 2.3, there can be multiple function forms for the MDS for which the nulls of nonlinearity specification, over-identification and no serial correlation in the first-difference equation are not rejected in a proper test. Fortunately, we arrive atonly one valid function form (denoted by GDP_n) for the model that excludes the interactive items. For the models that includes the interactive items, we also arrive at only one valid model GDP_s. To observe how interaction items affect the function forms of level and changes in MDS, let GDP_m denote the model in which we drop only the interaction items from the model GDP_s and keep other variables the same. The interactions between physical capital stock, working hours and MDS are dropped in the model GDP_s because they are individually and jointly insignificant. We find that GDP_m is also a valid function form. Using the Davidson-Mackinnon test, we find that the model GDP_s outperforms the models GDP_n and GDP_m, and that GDP_n outperforms GDP_m.

Our results show that the interactive items with capital stock and working hours are individually and jointly insignificant and that the significances of other explanatory variables are unaffected when they are dropped from the model. The interactive items that involve investment and human capital are individually and jointly significant. The six explanatory variables (investment, government spending, net export, inflation, working hours and capital stock) take the forms of current period and one-period lags in the model because they are individually or jointly significant; human capital and employment take the form of only the current period because their lags are individually insignificant, and joint insignificance is found for the forms of their current-period and one-period lags.

Table 2 below reports the robust system GMM estimates for the three models. The table lists only the estimates of MDS items, but the complete regression results can be found in Table A2 in the Appendix. MIS_t denotes the 5-year average mean income share at period *t*, MPS_t the 5-year average mean population share, I_t the 5-year average log investment *per capita*, and HC_t the 5-year average of log human capital *per capita*. The sign Δ before a variable refers to the 5-year average of the annual changes in that variable. The letter *D* before a variable means that the variable is demeaned by its panel mean, for instance, $D(I_{it}) = I_{it} - E(I_i)$. Here letter *E* before a variable refers to the panel average of the variable. The dependent variable is the 5-year average of log GDP *per capita* at period *t*.

Table 2 Robust System GMM Estimation Results

	GDP_{st}	GDP_{mt}	GDP_{nt}
ΔMIS_{t-1}	0.0081	0.0624	0.1110
$(\Delta MIS_{t-1})^2$	17.2612***	16.4585**	18.4987***
ΔMPS_{t-1}	0.1678	0.1149	0.1523
$(\Delta MPS_{t-1})^2$	-10.5034*	-9.0448	-10.8985*
MIS_t	0.2927**	0.2037*	
$[D(MIS_t)]^2$	-1.3608***	-0.2410	-5.0721**
$[D(MPS_t)]^2$			-1.1653
$D(MPS_t) * D(MIS_t)$			4.9481*

$D(I_t) * D(MPS_t)$	2.1489***		
$D(I_t) * D(MIS_t)$	-2.8377***		
$D(HC_t) * D(MPS_t)$	-9.1119***		
$D(HC_t) * D(MIS_t)$	4.9531		
No. of instruments	134	130	131
No. of observations	171	171	171

Notes: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$.

Source: Own elaboration.

We employ the Wald statistic to test for joint significance in the model GDP_s and find that the level changes in MIS and MPS are individually and jointly insignificant but that their quadratic items are individually very significant for the changes in MPS and MIS. The level MIS shows significant quadratic form as well. The linear and quadratic items of the changes in MPS are jointly significant at the 7.5% level. The function form of level MDS is found to be the quadratic form of MIS, which outperforms any other options using the tests that were discussed in Sub-section 2.3. Overall, in either changes or level forms, MIS shows more significant effects than does MPS.

The development and growth effect is $exp(0.0081 * \Delta MIS_{t-1} + 17.2612 * \Delta MIS_{t-1}{}^2)$ for ΔMIS_{t-1}, and it is $exp\,(0.1678 * \Delta MIS_{t-1} - 10.5034 * MIS_{t-1}{}^2)$ for ΔMPS_{t-1}; then, a combined growth effect of changes in MDS is the product of the two values, which can be extraction or expansion on the output $\widehat{GDP}_t$, depending on the signs and sizes of the changes in MDS. Table 3 below shows the growth effects of the changes in MDS at $t-1$, where changes in MDS at $t-1$ take the values 0.01, -0.01, and 0.

Table 3 Development and Growth Effects of Changes in MDS

Final effects on $\widehat{GDP}_t$		ΔMIS_{t-1}		
		-0.01	0	0.01
ΔMPS_{t-1}	-0.01	0.99891	**0.997272**	0.99908
	0	**1.001645**	**1**	**1.00181**
	0.01	1.002274	**1.000628**	1.00244

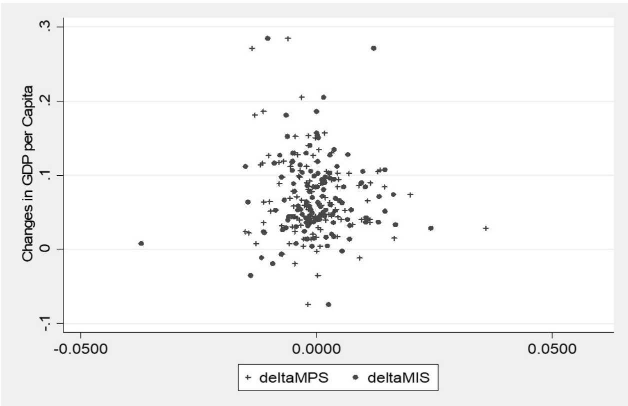
Notes: The highlighted column is also the growth effects of a change in MPS, and the highlighted row is the growth effects of a change in MIS. One standard deviation of the changes in MDS is approximately 1.3%; but 1% is applied here to simplify the calculation.

Source: Own elaboration.

Values less than one in the table indicate contraction effect, and expansion for values larger than one, on the output at period *t*. We find that the growth effects of an increase in MIS will always expand the economy and a decrease in MPS will always have negative effects on growth in this panel of countries.

For the model GDP_s, the sign of the quadratic item for the changes in MIS is positive, it is negative for the changes in MPS, and the estimations are statistically very significant. The average size of the top point for the changes in MPS is 0.83% for the entire panel dataset. Similarly, the average size of the bottom point for the changes of MIS is -0.024% for the entire panel dataset. Therefore, to maintain positive growth effects, the changes in MIS should be moving away from -0.024%, and the changes in MPS should be moving toward 0.83%.

We also find that the changes in MDS most likely correspond to a positive change in GDP in the next period. There are only 6 observations in the 5-year average panel data for which the changes in MDS correspond to a negative change in GDP; and the changes in MDS frequently change signs in the data. Thus, the growth effects of changes in MDS are not monotonic, and our empirical model GDP_s shows valid quadratic forms for the changes in MDS. Figure 4 below show the fitted changes in GDP_{s1} with the changes in MDS.



Source: Own elaboration.

Figure 4 The Fitted Changes in GDP with the Changes in MDS

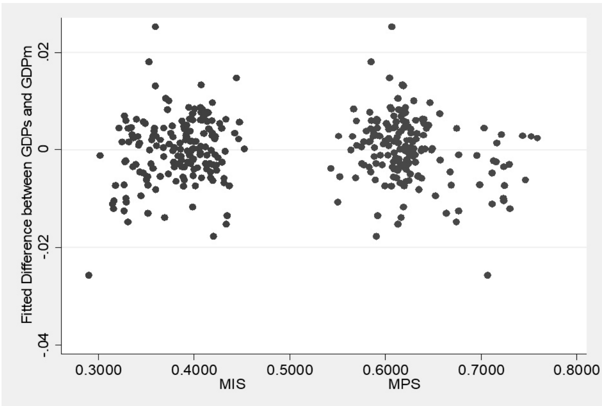
For the level MIS, there is an optimal size for an economy to optimize its development and growth. This optimal size of MIS depends on the levels of productive inputs in an economy, for instance, its human and physical capital development. If we ignore the interactive items to simplify the question, then the growth-optimal level of MIS is 48.2% for the entire panel of countries, approximately 10.8% larger than the mean MIS, 37.5%, in the panel dataset.

Lastly, Table 2 shows that the quadratic items of the changes in MPS and MIS show opposite signs in all three models and that the changes in MIS and level MIS show opposite signs. For instance, the changes in MIS shows U-shape effects, but those in MPS and level MIS show inverted U-shape effects. Thus, development and growth are an inverted U-shape function of the changes in MPS and level MIS and a U-shape function in the changes in MIS. These distinct effects of MIS and MPS cannot be described by any single summary measurement such as the Gini coefficient.

4.3 Effects of the Interactions between MDS and Input Factors

In comparing the estimation results of GDP_s with GDP_m and GDP_n, we find that the interactive items dramatically affect the choice of valid function forms for the level MDS but that the changes in MDS always show the same quadratic form whether the interactive items are included or not.

To check the effects of the interactive items in the model, we calculate the differences between their fitted values of GDP_s and GDP_m. We find that the two fitted values have the same mean, but the fitted GDP_m has a slightly smaller standard deviation (0.249) than the fitted GDP_s (0.2507); a difference of 0.7%. That is, the interaction between MDS and productive inputs slightly increases fluctuation in the output. Figure 5 below shows the fitted differences between GDP_s and GDP_m against MDS.



Source: Own elaboration.

Figure 5 Fitted Differences between GDP_s and GDP_m against MDS

The estimate of the interaction between human capital and MDS is $HC * (4.95 * MIS - 9.11 * MPS)$ and it is always negative. That is, the marginal growth effects of human capital have two parts: one directly from human capital itself, which can be positive, and the other from the interaction with MDS that is always negative. This negative growth effect of the interaction between human capital and MDS stems from the fact that MPS is always less than MIS, reflecting the existence of income inequality.

The estimate of the interaction between investment and MDS is $I * (2.15 * MPS - 2.84 * MIS)$ and it is negative only when $MIS > 0.76 * MPS$. Thus, the marginal growth effects of investment are most likely positive in underdeveloped countries in which mean population share is most likely sufficiently large compared with mean income share.

Our findings are interesting with respect to the economic effects of changes in income inequality proposed by Bernajee and Duflo (2003). First, the changes in MDS and level MDS are significant at 0.1% level, much higher than the significance levels of the Gini index items in Table 5 in Banerjee and Duflo (2003). The improved significance of the estimates may come from either a larger sample size or the different measurements of income inequality. Second, the MPS shows similar effects to those of the Gini coefficient, as found in Banerjee and Duflo (2003), but the MIS presents the opposite effects. Third, the MDS may give clearer policy implications than the Gini index because it is difficult to design a policy that targets changes in the Gini index and the corresponding goal of growth, but this is possible to implement this if the MPS are MIS are targeted.

We now reach the following conclusion. Using robust system GMM estimation on the one-period lagged dynamic estimator, the MDS significantly affects development and growth in three ways: the level MDS, the changes in MDS at last period and the interactive items of MDS with productive inputs. Current GDP *per capita* is an inverted U-shape function of the changes in MPS at the last period and MIS at the current period; and it is also a U-shape function of the changes in MIS at the last period. Thus, the marginal effects of MDS on development and growth can be either positive or negative depending on the MDS and on the macroeconomic and institutional framework captured by the interactive items in the model. A change in MDS would most likely correspond to a positive change in GDP *per capita* in the panel data, but the interaction between income inequality and productive input may increase fluctuations in growth.

These findings may lead us to the conjecture that the profile of income distribution matters in explaining its economic effects and that missing interactive items may negatively affect the analysis as well.

4.4 Policy Implication

There are important policy implications with the MDS. The measurement describes income inequality using two dimensions, MIS and MPS, to tell how many people are relatively poor and how poor they are at the mean income level. Thus, policy makers are able to choose which dimension of the two to target in promoting development and growth. A growth-optimal policy on income distribution must consider the levels of and changes in MDS, human capital, and investment levels and even particular combinations of these factors. Therefore, there exists a large pool of factor income distributional tools for policy makers to maintain their growth targets.

Specifically, the empirical analysis shows that growth-optimal policies with respect to income distribution require relatively small changes in MPS, which moves toward a particular point, and relatively large changes in MIS, which moves away from another particular point. The exact sizes of the changes in MDS depend on the macroeconomic and institutional framework, but there is an optimal relative size (48.2%) for MIS to promote growth in the entire panel of countries.

5. Concluding Remarks

We measure income inequality by MDS and allow interactive terms between the MDS and productive factors in a dynamic panel model. We apply robust system estimation to a one-period lag Arellano and Bond estimator that is heteroskedastic and serially correlated, and we use the same method to test the nonlinearity specification for function forms of income inequality, choosing the more powerful one of two optional and valid models. Then, we find that the levels of and changes in MDS show significant and non-monotonic effects on development and growth that can be either positive or negative depending on the levels of and changes in MDS and the interactions between productive inputs and MDS. Growth fluctuations may also come from the interactions between income inequality and productive inputs. Missing interactive items in the model may lead an incorrect choice of function forms for MDS and the analysis.

The MIS and MPS present opposite effects on growth. In particular, the MIS presents more significant effects than the MPS; relatively small changes in MPS or large changes in MIS lead to positive effects in growth. These different properties of the MDS from the Gini coefficient have advanced our understanding about income inequality. Specifically, introducing MDS adds a new way to interpret the growth and development effects of income inequality.

The optimal number of instruments and specification tests with instruments estimators for the Arellano and Bond system estimator are related issues for the dynamic model. This system estimation is consistent, but it may not be accurate because the estimation is sensitive to the number of instruments and our dataset is still insufficiently large. These issues demand additional work to obtain an unbiased estimation, and the economic effects of income inequality may require further comparison for different measurements.

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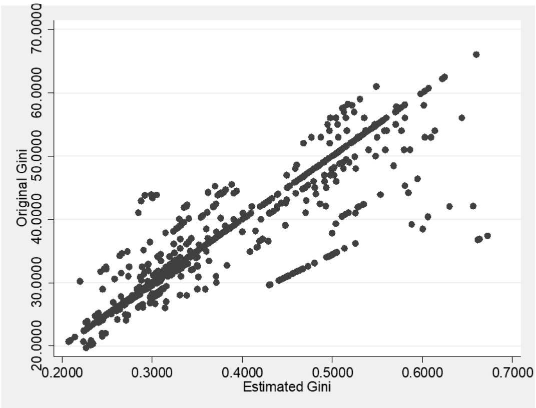
Appendix

Table A1 Five-Year Average Data Summary

Variable	Obs	Mean	Std. dev.	Min	Max
Gini	329	0.3783	0.1056	0.2115	0.6511
MPS	329	0.6458	0.0511	0.5430	0.7879
MIS	329	0.3748	0.0389	0.2710	0.4616
∆Gini	308	-0.0006	0.0186	-0.1224	0.0840
∆MPS	308	0.0000	0.0134	-0.0589	0.0590
∆MIS	308	0.0004	0.0128	-0.0712	0.0610
GDP	329	4.0210	0.4360	2.8803	4.8929
Capital stock	329	4.4624	0.4920	3.0840	5.3111
Investment	329	3.3378	0.5162	1.6900	4.2678
GOV	329	3.2496	0.4675	1.8407	3.9299
XM	329	3.6651	0.6957	1.6660	5.1541
Human capital	322	0.4114	0.0910	0.0605	0.5554
Working hour	230	2.0984	0.6483	0.7507	3.7887
Employment	328	0.4199	0.0712	0.2440	0.7180

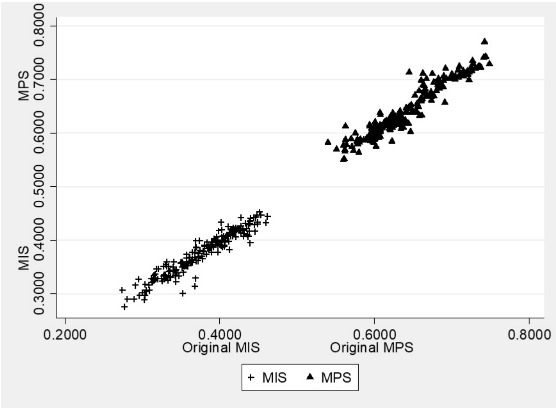
Notes: The data are non-overlapping 5-year averages.

Source: Own elaboration.



Source: Own elaboration.

Figure A1 The Original and Estimated Gini Indices



Source: Own elaboration.

Figure A2 The Original and Estimated MDS

Table A2 Complete Estimation Results

	GDP_{st}	GDP_{mt}	GDP_{nt}
GDP_{t-1}	0.7969***	0.7835***	0.7805***
GDP_{t-1}^2	-0.1649***	-0.1322**	-0.1243**
ΔMIS_{t-1}	0.0081	0.0624	0.1110
$(\Delta MIS_{t-1})^2$	17.2612***	16.4585**	18.4987***
ΔMPS_{t-1}	0.1678	0.1149	0.1523
$(\Delta MPS_{t-1})^2$	-10.5034*	-9.0448	-10.8985*
MIS_t	0.2927**	0.2037*	
$[D(MIS_t)]^2$	-1.3608***	-0.2410	-5.0721**
$[D(MPS_t)]^2$			-1.1653
$D(MPS_t) * D(MIS_t)$			4.9481*
$D(I_t) * D(MPS_t)$	2.1489***		
$D(I_t) * D(MIS_t)$	-2.8377***		
$D(HC_t) * D(MPS_t)$	-9.1119***		
$D(HC_t) * D(MIS_t)$	4.9531		
I_t	0.2767***	0.2399***	0.2446***
I_{t-1}	-0.2073***	-0.2151***	-0.2157***
GOV_t	0.1861***	0.1825***	0.1935***
GOV_{t-1}	-0.2562***	-0.2492***	-0.2396***
XM_t	0.1529***	0.1523***	0.1607***
XM_{t-1}	-0.0913***	-0.0721***	-0.0772***
PAI_t	-0.2229***	-0.2278***	-0.2445**
PAI_{t-1}	-0.0259	-0.0393	-0.0526*
$Capital\ stock_t$	0.1992*	0.3218**	0.2822**

$Capital\ stock_{t-1}$	-0.1548*	-0.2454**	-0.2282**
$Working\ hour_t$	-0.1585	-0.1858	-0.1804
$Working\ hour_{t-1}$	0.1428	0.1668	0.1633
$Human\ capital_t$	0.1050	0.0763	0.0491
$Employment\ rate_t$	0.1228*	0.1102	0.1436*
Constant	0.2311*	0.2662**	0.3595***

Notes: * $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$. The observation size is 171. The number of instrument is 138, 130, and 131 for the model GDP_{st}, GDP_{mt}, GDP_{st}, respectively.

Source: Own elaboration.