Privatization, Pollution, and Welfare in a Mixed Differentiated Duopoly

Summary: This paper investigates the impacts exerted by the residents’ environmental preference on privatization in a mixed differentiated duopoly. We assume that the production will generate environmental pollution, which causes an extra cost that the private firm does not bear but residents with the environmental preference have to tolerate. The government pursues to maximize social welfare. We find that the residents’ environmental preference has significant impacts on privatization. When residents pay more attention to environmental pollution, privatization cannot induce more social welfare, and thus the government will choose not to privatize the public firm. The degree of substitution of products will affect the results when the value of the residents’ environmental preference is given, but the order of the firms’ moves will not change the results.

Key words: Environmental preference, Privatization, Differentiated duopoly, Social welfare.

JEL: H21, L33, Q58.

The issue about privatization is a hot academic topic. Since the 1980s the reform in the former socialistic countries and the privatization waves in the western countries had occurred simultaneously. The process of privatization had been accelerated because politicians declared that privatization could make the economy more efficient and induce more social welfare, and the mainstream economics also stood strongly by them. So we have witnessed the prevalence of privatization.

Indeed privatization can induce more social welfare in some economies like Western European countries, Czech Republic and China, and make the economies in these countries more competitive and efficient. But privatization has yet made some countries suffer from the deterioration in social welfare, like Russia and some Eastern European countries. Even in those countries where privatization has succeeded in improving the economy, many problems have emerged during the course of privatization. For example, some transitional Eastern European countries experienced a strong transitional recession in the early 1990s. Among them, the widening income gap and the serious environmental pollution problem are the most salient examples.

This paper tries to explore the relation between environmental pollution and privatization. For that purpose, we build game theoretical models under the framework of the mixed differentiated duopoly. We find that if residents do not pay more attention to environmental pollution, privatization can increase social welfare, and that the possibility of increasing social welfare after privatization will decrease as the degree of
substitution of products increases. Although the impacts of privatization on social welfare under Cournot competition and Stackelberg competition are similar, social welfare after privatization under Stackelberg competition is higher than that under Cournot competition only when the environmental preference is weak enough. At the same time, as the degree of substitution increases, the possibility of higher social welfare under Stackelberg competition than that under Cournot competition after privatization will decrease.

This paper is organized as follows. In Section 1, we review the related literature and give the strategy of this paper. In Section 2, we explore the effects of privatization and the effects that the environmental preference exerts on privatization in the Cournot game and the Stackelberg game, respectively. In Section 3, we compare the results under Cournot competition and Stackelberg competition, especially in terms of social welfare. In Section 4, we discuss the limitations arising from our approach and model. In Section 5, some concluding remarks are provided.

1. Literature Review and Analytical Framework

This section is divided into three parts. In Subsection 1.1, we review the literature on the relation between privatization and social welfare. In Subsection 1.2, we survey the literature on the relation between environmental pollution and privatization. In Subsection 1.3, we describe the strategy of this paper.

1.1 The Relation between Privatization and Social Welfare

Privatization is a useful policy tool for the government to influence economic activities and social welfare (see Giovanni de Fraja and Flavio Delbono 1990). As for exploring the relation between privatization and social welfare, there is abundant studies (e.g., De Fraja and Delbono 1989; Toshihiro Matsumura 1998; Kenneth Fjell and John S. Heywood 2004; Juan Carlos Bárcena-Ruiz and María Begoña Garzón 2006; Matsumura, Noriaki Matsushima, and Ikuo Ishibashi 2009; Bárcena-Ruiz 2012), and many of them are typically representative. De Fraja and Delbono (1989) prove that the number of the firms and the modes of competition (e.g., Cournot competition or Stackelberg competition) have important effects on the relation between privatization and social welfare. Matsumura (1998) finds that there is a trade-off in the privatization process and there exists an optimal level of partial privatization. Fjell and Heywood (2004) investigate how the government uses the subsidy as a policy tool to influence privatization and they show that the order of the firm’s moves generates important impacts on the results. Matsumura, Matsushima, and Ishibashi (2009) discuss the effect of the entry of foreign firms on the relation between privatization and social welfare. Bárcena-Ruiz (2012) explores privatization in the case where the efficiency of the public firm is the same as that of private firms.

The literature in this direction usually doesn’t take environmental pollution into account. With a special view to environmental pollution, this paper tries to explore the impacts exerted by the residents’ environmental preference on privatization in a mixed differentiated duopoly.
1.2 The Relation between Environmental Pollution and Privatization

The literature on the relation between environmental pollution and privatization is also abundant. Dietrich Earnhart and Lubomir Lizal (2006) empirically analyze the impacts of ownership structure on the environmental performance of the corporations in a transition economy. They conclude that private ownership will increase absolute emissions relative to state ownership. Shuichi Ohori (2006) proves that with the consideration of environmental pollution the optimal level of privatization is partial and depends on the disposal cost of environmental pollution inversely. Debabrata Talukdar and Craig M. Meisner (2001) explore whether privatization helps the environment or not by empirical analysis. They show that in the long-term privatization will help the environment. However, here we argue that there are some controversial problems in their paper. For example, they choose the emission of carbon dioxide as a dependent variable, but we know that carbon dioxide itself is not pollution at all in the environmental science. Although the literature in this direction is abundant, there are still some points that we can explore further. The aforementioned literature mostly analyzes the effects on environmental pollution exerted by product differentiation, ownership structure, and privatization. In this paper, we aim at investigating the effects of environmental pollution on privatization. There is a tiny literature on environmental pollution considering the effects of the order of the firms’ moves, and the exception is Bárcena-Ruiz (2006), who deeply discusses such effects under environmental policies.

This paper contributes to the existing literature in the following respect. We attach great importance to the residents’ environmental preference and focus on its impact on the decision of privatization in a differentiated oligopoly under different modes of competition.

1.3 The Strategy of This Paper

This paper tries to find a new mechanism for privatization and to analyze the decision of the privatization policy from a new perspective. Environmental pollution is generated by the firms in the course of production. There are many studies investigating environmental pollution through the framework of the mixed oligopoly. Rupayan Pal (2012) explores the effects of product differentiation and managerial delegation on the optimal emission tax rate, pollution and social welfare in a differentiated oligopoly. Bárcena-Ruiz and Maria Luz Campo (2012) analyze the impacts of cross-ownership in the firms of different countries on the pollution taxes set by different countries in a mixed oligopoly. Oliwia Kurtyka and Philippe Mahenc (2011) provide an original insight on the function of environmental taxes in a Bertrand differentiated duopoly. Hassan Benchekroun and Amrita Ray Chaudhuri (2011) prove that in a dynamic oligopoly the Markovian tax can induce stable cartelization in the industry. Chiara Lombardini-Riipinen (2005) investigates the optimal environmental tax policy when consumers prefer the environmentally friendlier commodities and the commodities are differentiated in the environmental quality vertically. To the best of our knowledge, the aforementioned literature does not take the residents’ environmental preference into account. Although Jiancai Pi, Li Yang, and Yu Zhou (2013) consider the residents’ environmental preference, they don’t adopt the framework of mixed differentiated
We introduce environmental pollution into the social welfare function. In such a function, the weight assigned to pollution is used to denote the degree of attention that residents pay to the pollution problem, which is called the environmental preference (see Pi, Yang, and Zhou 2013). The larger the value of the environmental preference is, the more attention residents pay to environmental pollution, and the more seriously residents suffer from the same level of environmental pollution. The government is assumed to be benevolent in the sense that it pursues the maximization of the residents’ welfare. That is, we can think that the policy decision of the government is dependent on the votes of residents and is affected by the residents’ environmental preference. The conventional wisdom holds that in the mixed duopoly framework after privatization the consumer surplus will be reduced and the profits of the two firms will be increased because the output will be reduced and the increased profits will overwhelm the decreased consumer surplus. But in our model, the things are not so simple because a negative externality is introduced into the duopoly. The production process will generate environmental pollution, but the firms do not need to bear any extra cost. Here we don’t consider the case where the firms bear the cost to abate pollution or the firms are taxed for their pollution emission that has been deeply discussed as the environmental policy in the existing literature. In our paper, in the absence of the environmental policy the output of public firm can act as a regulatory device to control pollution. The public firm can control pollution by controlling its output. As we know, the production decision of the public firm is manipulated by the government, and the government cares for the pollution problem. The private firm does not need to be concerned with the pollution problem, although it must take the output of the public firm into account. Thus, if the residents’ environmental preference is appropriate, the production of the mixed duopoly may be more efficient than that after privatization, and furthermore privatization perhaps cannot increase social welfare. We also analyze the impacts of the degree of substitution and the order of the firms’ moves. We find that the residents’ environmental preference generates a significant impact on the decision of privatization, and that the impact is affected by the degree of substitution.

2. Theoretical Model

We assume that there exist a private firm (denoted by firm 1) and a public firm (denoted by firm 0). The products produced by firms 0 and 1 are horizontally differentiated and partially substitutable. The demand function is assumed to be linear (see e.g., Pal 2012), which is given by:

\[ p_i = 1 - q_i - bq_j, \tag{1} \]

where \( i, j = 0, 1, i \neq j \), and \( b \in [0,1] \) measures the degree of substitution between different products; \( q_0 \) is the output of the public firm and \( q_1 \) is the output of the private firm.
So total consumer surplus is:

\[ CS = \frac{1}{2} (q_0^2 + 2bq_0q_1 + q_1^2). \]  

(2)

We assume the cost functions of the two firms are quadratic, which can be described by:

\[ c(q_i) = \frac{1}{2} q_i^2, \]  

(3)

where \( c(q_i) \) is the total cost of firm \( i \). Because the entry problem of the firms is not considered in this paper, the fixed cost is set to be zero without loss of generality.

We assume that both the public firm and the private firm produce pollution. Even if the firms can choose to abate pollution, there still exists net emission to the environment. We suppose that one unit of output brings about one unit of net emission. Thus, we have \( E = q_0 + q_1 \), where \( E \) is the total net pollutant discharge level.

The profits of the public firm and the private firm are given by:

\[ \pi_0 = p_0q_0 - \frac{1}{2} q_0^2, \]  

(4)

\[ \pi_1 = p_1q_1 - \frac{1}{2} q_1^2. \]  

(5)

The goal of the private firm is to maximize its own profit, and the goal of the public firm is to maximize social welfare. Since residents dislike environment pollution, social welfare in our paper must include the disutility of pollution. So the social welfare function can be expressed as:

\[ W = CS + PS - \lambda (q_0 + q_1), \]  

(6)

where \( CS \) and \( PS \) are the consumer surplus and the producer surplus, respectively. The parameter \( \lambda \) measures the environmental preference of residents. The bigger the value of \( \lambda \) is, the more the degree that residents care about the environment. If residents don’t care about the environment at all, \( \lambda \) is set equal to 0.

Then we will discuss the effect of the environment preference on privatization under Cournot competition and Stackelberg competition, respectively.

2.1 Cournot Competition

We will firstly discuss the equilibrium values of Cournot competition before privatization. The public firm and the private firm choose their quantities simultaneously. We assume that \( \lambda \in [0, \frac{4+b}{2(3+b)}] \) in order to make our analysis interesting.

Here, it should be noted that the subscript \( c \) denotes Cournot competition, and the superscripts \( ex \) and \( pri \) stand for before and after privatization, respectively.
We firstly discuss the equilibrium values in the mixed duopoly before privatization. In this case, it is easy to obtain:

\[ q_{0c}^{ex} = \frac{3 - b - 3\lambda}{6 - b^2}, \]  
(7-1)

\[ q_{1c}^{ex} = \frac{2 - b + b\lambda}{6 - b^2}, \]  
(7-2)

\[ Q_{c}^{ex} = \frac{5 - 2b - (3 - b)\lambda}{6 - b^2}, \]  
(7-3)

\[ CS_{c}^{ex} = \frac{\left[ (3-b)(3+3b - 2b^2) + (2-b^2) \right]}{2(6-b^2)^2}, \]  
(7-4)

\[ \pi_{0c}^{ex} = \frac{[3-b+(9-2b^2)\lambda][3-b-3\lambda]}{2(6-b^2)^2}, \]  
(7-5)

\[ \pi_{1c}^{ex} = \frac{3(2-b+b\lambda)^2}{2(6-b^2)^2}, \]  
(7-6)

\[ W_{c}^{ex} = \frac{\left[ (9 - 6b - b^2 + b^3)\lambda^2 \right]}{(6-b^2)^2}, \]  
(7-7)

Now let us discuss the equilibrium values of Cournot competition after privatization. In this situation, it is easy to get:

\[ q_{0c}^{pri} = \frac{1}{3+b}, \]  
(8-1)

\[ q_{1c}^{pri} = \frac{1}{3+b}, \]  
(8-2)

\[ Q_{c}^{pri} = \frac{2}{3+b}, \]  
(8-3)

\[ CS_{c}^{pri} = \frac{1+b}{(3+b)^2}, \]  
(8-4)

\[ \pi_{0c}^{pri} = \frac{3}{2(3+b)^2}, \]  
(8-5)
\[ \pi_{1c}^{pri} = \frac{3}{2(3+b)^2}, \]  
\[ W^c_{pri} = \frac{4+b - 2(3+b)\lambda}{(3+b)^2}. \]

After privatization, both firms are private ones and pursue the maximization of their own profits. Hence, their outputs and profits are the same and irrelevant to the residents’ environmental preference, so are the consumer surplus and the producer surplus. However, because residents suffer from environmental pollution, an increase in the residents’ environmental preference will decrease social welfare.

Comparing the results in Cournot game before and after privatization, we can find that when \( \lambda = \frac{1}{3+b} \) privatization has no socioeconomic impacts. Otherwise, privatization has the following economic effects that can be summarized to Propositions 1-4.

**Proposition 1:** (i) If the environmental preference is weak enough, privatization will decrease the output of the public firm and increase that of the private firm; and if the environmental preference is strong enough, privatization will enhance the output of the public firm and reduce that of the private firm. (ii) As the differentiation of products diminishes, privatization will lead more easily to a rise of the output of the public firm and a drop of the output of the private firm.

**Proof:** If \( 0 < \lambda < \frac{1}{3+b} \), then \( q_{0c}^\text{ex} - q_{0c}^{pri} > 0 \) and \( q_{1c}^\text{ex} - q_{1c}^{pri} < 0 \). If \( \frac{1}{3+b} < \lambda < \frac{4+b}{2(3+b)} \), then \( q_{0c}^\text{ex} - q_{0c}^{pri} < 0 \) and \( q_{1c}^\text{ex} - q_{1c}^{pri} > 0 \). As \( b \) increases (i.e., the degree of the differentiation of products becomes small), the interval of \( \lambda \) will be shortened in the respect that privatization can decrease the output of the public firm and increase that of the private firm, and the interval of \( \lambda \) will be lengthened in the respect that privatization can raise the output of the public firm and reduce that of the private firm. •

**Proposition 2:** (a) If the environmental preference is weak enough or strong enough, privatization will enhance the public firm’s profit; and if the environmental preference is moderate, privatization will decrease the public firm’s profit. (b) If the environmental preference is weak enough, privatization will increase the profit of the private firm; and if the environmental preference is strong enough, privatization will decrease the profit of the private firm. (c) As the differentiation of products diminishes, privatization will tend more easily to reduce the public firm’s profit and the private firm’s profit.

**Proof:** If \( 0 < \lambda < \frac{27-18b^2 + 2b^4}{81 + 27b - 18b^2 - 6b^3} \) and \( \frac{1}{3+b} < \lambda < \frac{4+b}{2(3+b)} \), then \( \pi_{0c}^{\text{ex}} - \pi_{0c}^{\text{pri}} < 0 \). If \( \frac{27-18b^2 + 2b^4}{81 + 27b - 18b^2 - 6b^3} < \lambda < \frac{1}{3+b} \), then \( \pi_{0c}^{\text{ex}} - \pi_{0c}^{\text{pri}} > 0 \).
If $0 < \lambda < \frac{1}{3+b}$, then $\pi_{lc}^{ex} - \pi_{lc}^{pri} < 0$. If $\frac{1}{3+b} < \lambda < \frac{4+b}{2(3+b)}$, then $\pi_{lc}^{ex} - \pi_{lc}^{pri} > 0$. As $b$ increases, the interval of $\lambda$ will be narrowed down in the respect that privatization can enhance the profits of the public firm and the private firm, and the interval of $\lambda$ will be widened in the respect that privatization can reduce the profits of the public firm and the private firm.

**Proposition 3:** (1) If the environmental preference is weak enough, privatization will reduce the consumer surplus; and if the environmental preference is strong enough, privatization will raise the consumer surplus. (2) If the environmental preference is weak enough or strong enough, privatization will enhance the producer surplus; and if the environmental preference is moderate, privatization will reduce the producer surplus. (3) As the differentiation of products diminishes, privatization will tend more easily to increase the consumer surplus and decrease the producer surplus.

**Proof:** If $0 < \lambda < \frac{1}{3+b}$, then $CS_{c}^{ex} - CS_{c}^{pri} > 0$. If $\frac{1}{3+b} < \lambda < \frac{4+b}{2(3+b)}$, then $CS_{c}^{ex} - CS_{c}^{pri} < 0$. If $0 < \lambda < \frac{1}{3+b}$ and $\frac{27+36b-21b^2-6b^3+2b^4}{81+27b-27b^2-9b^3} < \lambda < \frac{4+b}{2(3+b)}$, then $PS_{c}^{ex} - PS_{c}^{pri} < 0$; and if $\frac{1}{3+b} < \lambda < \frac{27+36b-21b^2-6b^3+2b^4}{81+27b-27b^2-9b^3}$, then $PS_{c}^{ex} - PS_{c}^{pri} > 0$. As $b$ increases, the interval of $\lambda$ will be shortened in the respect that privatization can reduce the consumer surplus and raise the producer surplus, and the interval of $\lambda$ will be lengthened in the respect that privatization can increase the consumer surplus and decrease the producer surplus.

**Proposition 4:** (i) If the environmental preference is weak enough, privatization will increase social welfare; and if the environmental preference is strong enough, privatization will reduce social welfare. (ii) As the differentiation of products diminishes, privatization will tend more easily to decrease social welfare.

**Proof:** If $0 < \lambda < \frac{1}{3+b}$, then $W_{c}^{ex} - W_{c}^{pri} < 0$. If $\frac{1}{3+b} < \lambda < \frac{4+b}{2(3+b)}$, then $W_{c}^{ex} - W_{c}^{pri} > 0$. As $b$ increases, the interval of $\lambda$ will be narrowed down in the respect that privatization can enhance social welfare.

The intuition behind these propositions is straightforward. The public firm must consider social welfare that is affected by environmental pollution, so it has to face a trade-off between increasing its output that may generate more pollution and decreasing its output that may produce less pollution. If residents care less about the environment, that is, their environmental preference is relatively weak (i.e., $0 < \lambda < \frac{1}{3+b}$), the public firm will care more about the consumer surplus. Thus, the output of the public firm will be relatively higher compared with that after privatization, but the private firm will produce less output than that after privatization. An increase in the public firm’s output can exert a positive impact on the consumer surplus but a negative impact on the producer surplus and the environment. If the residents’ environmental preference is strong enough, the public firm will care more about the consumer surplus than about the environmental pollution, so it will produce less output than that after privatization.
preference is relatively weak (i.e., $0 < \lambda < \frac{1}{3+b}$), privatization can improve social welfare because an increase in the producer surplus and the environmental quality after privatization induced by a decrease in the output outweighs a decrease in the consumer surplus. If residents care more about the environment, that is, their environmental preference is relatively strong (i.e., $\frac{1}{3+b} < \lambda < \frac{4+b}{2(3+b)}$), the public firm will care more about the environmental quality. So the public firm will decrease its output compared with that after privatization, but the private firm will increase its output in order to earn more profit. The consequence is that the private firm will earn more profit than that after privatization, and that the public firm will suffer from its reduced output compared with that after privatization. Thus, as the residents’ environmental preference increases, the producer surplus will be larger than that after privatization firstly but then smaller than that after privatization. Privatization can decrease social welfare because the decrease in the environmental quality outweighs the increase in the consumer surplus, although the impact on the producer surplus is ambiguous.

2.2 Stackelberg Competition

In this case, we will change the order of the firms’ moves. In the first stage, the public firm determines $q_0$ to maximize social welfare. In the second stage, knowing the value of $q_0$, the private firm chooses $q_1$ to maximize its profit. In the two-stage game, the public firm can be seen as a leader, whose leadership may stem from its political connection or technological advantage. In order to get an inner solution, we assume that

$$\lambda \in \left[0, \frac{36-15b-b^2}{3(18-6b-b^2)}\right].$$

The equilibrium values in the mixed duopoly under Stackelberg competition are given as follows:

$$q_{0e} = \frac{9 - 4b - 3(3-b)\lambda}{18 - 4b^2}, \quad (9-1)$$

$$q_{1e} = \frac{6 - 3b + (3-b)b\lambda}{18 - 4b^2}, \quad (9-2)$$

$$CS_{se} = \left[\frac{(81 - 54b - 36b^2 + 30b^3 - 5b^4)\lambda^2}{-(162 - 54b - 90b^2 + 54b^3 - 8b^4)\lambda + 117 - 77b^2 + 24b^3} \right] \frac{8(9 - 2b^2)^2}{9(9 - 2b^2)^2}, \quad (9-3)$$

$$\pi_{0e} = \frac{1 + (3-b)\lambda}{} \frac{[9 - 4b - 3(3-b)\lambda]}{4(18 - 4b^2)}, \quad (9-4)$$
Now let us discuss the equilibrium values in the duopoly under Stackelberg competition after privatization. In this case, it is not hard to obtain:

\[
q_{0s}^{pri} = \frac{3 - b}{9 - 2b^2}, \tag{10-1}
\]

\[
q_{1s}^{pri} = \frac{9 - 3b - b^2}{3(9 - 2b^2)}, \tag{10-2}
\]

\[
CS_s^{pri} = \frac{162 + 54b - 108b^2 + 6b^3 + 7b^4}{18(9 - 2b^2)^2}, \tag{10-3}
\]

\[
\pi_{0s}^{pri} = \frac{(3 - b)^2}{6(9 - 2b^2)^2}, \tag{10-4}
\]

\[
\pi_{1s}^{pri} = \frac{(9 - 3b - b^2)^2}{6(9 - 2b^2)^2}, \tag{10-5}
\]

\[
W_s^{pri} = \frac{36 - 15b - b^2 - 3(18 - 6b - b^2)\lambda}{9(9 - 2b^2)}. \tag{10-6}
\]

The analysis of the equilibrium values after privatization is similar to that under Cournot competition after privatization. After privatization, the ex-public firm changes to pursue the maximization of its own profit. So the outputs of the two firms are both irrelevant to the residents’ environmental preference. Thus, the change of the residents’ environmental preference only affects social welfare. Stronger environmental preference means that residents will pay more attention to environmental pollution, and thus, ceteris paribus, social welfare will fall.

Now we will compare the equilibrium values after privatization with those before privatization. If \( \lambda = \frac{3 - 2b}{9 - 3b} \), privatization has no economic impacts. Otherwise, privatization has the following economic effects which are summarized to Propositions 5-8.

**Proposition 5:** (a) If the environmental preference is weak enough, privatization will decrease the output of the public firm and increase that of the private firm; and if the environmental preference is strong enough, privatization will enhance the output of the public firm and reduce that of the private firm. (b) As the differentiation...
of products diminishes, privatization will tend more easily to raise the output of the public firm and reduce the output of the private firm.

**Proof:** If $0 < \lambda < \frac{3-2b}{9-3b}$, then $q_{0s}^{ex} - q_{0s}^{pri} > 0$ and $q_{1s}^{ex} - q_{1s}^{pri} < 0$. If $\frac{3-2b}{9-3b} < \lambda < \frac{36-15b-b^2}{3(18-6b-b^2)}$, then $q_{0s}^{ex} - q_{0s}^{pri} < 0$ and $q_{1s}^{ex} - q_{1s}^{pri} > 0$. As $b$ increases, the interval of $\lambda$ will be narrowed down in the respect that privatization can reduce the output of the public firm and raise the output of the private firm, and the interval of $\lambda$ will be widened in the respect that privatization can increase the output of the public firm and decrease the output of the private firm.

**Proposition 6:** (1) No matter how strong the environmental preference is, privatization will not reduce the profit of the public firm. (2) If the environmental preference is weak enough, privatization will raise the profit of the private firm; and if the environmental preference is strong enough, privatization will reduce the profit of the private firm. (3) As the differentiation of products diminishes, privatization will tend more easily to decrease the profit of the private firm.

**Proof:** If $\lambda \neq \frac{3-2b}{9-3b}$, then $\pi_{0s}^{ex} - \pi_{0s}^{pri} < 0$. If $0 < \lambda < \frac{3-2b}{9-3b}$, then $\pi_{1s}^{ex} - \pi_{1s}^{pri} < 0$. If $\frac{3-2b}{9-3b} < \lambda < \frac{36-15b-b^2}{3(18-6b-b^2)}$, then $\pi_{1s}^{ex} - \pi_{1s}^{pri} > 0$. As $b$ increases, the interval of $\lambda$ will be shortened in the respect that privatization can raise the profit of the private firm, and the interval of $\lambda$ will be lengthened in the respect that privatization can reduce the profit of the private firm.

**Proposition 7:** (i) If the environmental preference is weak enough, privatization will reduce the consumer surplus; and if the environmental preference is strong enough, privatization will raise the consumer surplus. (ii) When $b \in [0,0.8)$, if the environmental preference is weak enough or strong enough, privatization will enhance the producer surplus; and if the environmental preference is moderate, privatization will decrease the producer surplus. When $b \in [0.8,1)$, if the environmental preference is weak enough, privatization will increase the producer surplus; and if the environmental preference is strong enough, privatization will decrease the producer surplus. (iii) As the differentiation of products diminishes, privatization will tend more easily to enhance the consumer surplus and reduce the producer surplus.

**Proof:** If $0 < \lambda < \frac{3-2b}{9-3b}$, then $CS_{s}^{ex} - CS_{s}^{pri} > 0$. If $\frac{3-2b}{9-3b} < \lambda < \frac{36-15b-b^2}{3(18-6b-b^2)}$, then $CS_{s}^{ex} - CS_{s}^{pri} < 0$. When $b \in [0,0.8)$, if $0 < \lambda < \frac{3-2b}{9-3b}$ and $\frac{27+18b-21b^2+2b^3}{81-27b-27b^2+9b^3} < \lambda < \frac{36-15b-b^2}{3(18-6b-b^2)}$, then $PS_{s}^{ex} - PS_{s}^{pri} < 0$; and if $\frac{3-2b}{9-3b} < \lambda < \frac{27+18b-21b^2+2b^3}{81-27b-27b^2+9b^3}$, then $PS_{s}^{ex} - PS_{s}^{pri} > 0$. When $b \in [0.8,1)$, if
0 < \lambda < \frac{3 - 2b}{9 - 3b}, \text{ then } P_{s}^{ex} - P_{s}^{pri} < 0; \text{ and if } \frac{3 - 2b}{9 - 3b} < \lambda < \frac{36 - 15b - b^2}{3(18 - 6b - b^2)}, \text{ then } P_{s}^{ex} - P_{s}^{pri} > 0. \text{ As } b \text{ increases, the interval of } \lambda \text{ will be narrowed down in the respect that privatization can reduce the consumer surplus, and the interval of } \lambda \text{ will be widened in the respect that privatization can enhance the consumer surplus. At the same time, as } b \text{ increases, the interval of } \lambda \text{ will be shortened in the respect that privatization can raise the producer surplus, and the interval of } \lambda \text{ will be lengthened in the respect that privatization can reduce the producer surplus.} \]

**Proposition 8:** (a) If the environmental preference is weak enough, privatization will raise social welfare; and if the environmental preference is strong enough, privatization will reduce social welfare. (b) As the differentiation of products diminishes, privatization will tend more easily to decrease social welfare.

**Proof:** If \(0 < \lambda < \frac{3 - 2b}{9 - 3b}\), then \(W_{s}^{ex} - W_{s}^{pri} < 0\). If \(\frac{3 - 2b}{9 - 3b} < \lambda < \frac{36 - 15b - b^2}{3(18 - 6b - b^2)}\), then \(W_{s}^{ex} - W_{s}^{pri} > 0\). As \(b\) increases, the interval of \(\lambda\) will be shortened in the respect that privatization can enhance social welfare, and the interval of \(\lambda\) will be lengthened in the respect that privatization can reduce social welfare.

The economic intuition behind Propositions 5-8 under Stackelberg competition is similar to those behind Propositions 1-4 under Cournot competition. However, the difference is that privatization under Cournot competition will not decrease the profit of the public firm. The welfare consequence under Stackelberg competition is also similar to that under Cournot competition. When the residents’ environmental preference is not strong enough, privatization will increase social welfare because the positive impact of privatization outweighs the negative impact of privatization (i.e., environmental pollution). When the residents’ environmental preference is relatively strong, privatization will reduce social welfare because the negative impact of privatization (i.e., environmental pollution) outweighs the positive impact of privatization. Greatly different from the situation without the environmental pollution problem where privatization can always enhance social welfare, in the situation where the public firm must consider the residents’ environmental preference, privatization may improve or deteriorate social welfare conditional on the magnitude of the residents’ environmental preference. Just like propositions under Cournot competition, the impacts of the environmental preference on privatization under Stackelberg competition are also affected by the degree of substitution. The effects that the change of the degree of substitution exerts on privatization are almost the same as those in the propositions under Cournot competition and so are the economic intuitions behind them.

### 3. Discussion

In this section, in order to gain more insights, we will compare the results under Cournot competition with those under Stackelberg competition.

After privatization, the ex-public firm pursues the maximization of its own profit. The outputs of the two firms, the profits of the two firms, the consumer surplus and the producer surplus are irrelevant with environmental pollution.
The comparisons between social welfare under Cournot competition and that under Stackelberg competition is summarized by Proposition 9.

**Proposition 9:** (1) Before privatization, if \( \lambda \neq \frac{1}{3+b} \), social welfare under Stackelberg competition will be higher than that under Cournot competition. (ii) After privatization, if \( 0 < \lambda < \frac{b^2 + 3b - 9}{3b^2 - 27} \), Stackelberg competition will induce higher social welfare than Cournot competition; and if \( \frac{b^2 + 3b - 9}{3b^2 - 27} < \lambda < \frac{36 - 15b - b^2}{3(18 - 6b - b^2)} \), Cournot competition will induce higher social welfare than Stackelberg competition after privatization. (2) As the differentiation of products diminishes, Cournot competition will tend more easily to result in higher social welfare than Stackelberg competition after privatization.

From the microeconomics textbook, we know that Stackelberg competition will lead to higher social welfare and larger total output than Cournot competition if there are no externalities. So if the production generates environmental pollution and residents do not care about environmental pollution, pollution under Stackelberg competition will be more serious than that under Cournot competition. Thus, if the residents’ environmental preference is not very strong, social welfare under Stackelberg competition will be higher than that under Cournot competition. However, if the residents’ environmental preference is relatively strong, that is, residents care more about the environmental problem, the negative effect of more outputs will outweigh the positive effect of more outputs and Cournot competition will achieve higher social welfare than Stackelberg competition.

Comparing with Pi, Yang, and Zhou (2013), we can find that they stress the role of the market scale in determining the size of the critical value of the residents’ environmental preference when it is in terms of social welfare. However, this paper highlights the role of the degree of substitution between different products instead of the role of the market scale. In fact, we can find that the degree of substitution between different products plays a role that should not be neglected during the course of privatization.

4. Limitations Arising from Our Approach and Model

In this section, we will discuss some limitations arising from our approach and model. Firstly, similar to Pi, Yang, and Zhou (2013) and other related studies, this paper assumes that the firms’ cost functions are quadric and that environmental pollution is generated linearly in terms of the firms’ outputs. Although such assumptions can simplify our analyses, they also mask the real-world complexities of production and pollution. Secondly, this paper neglects the role of clean technologies and governmental regulation in reducing pollution. When residents pay more attention to environmental pollution, the government may adopt stricter environmental regulation and the firms may choose cleaner technology (see e.g., Maryam Asghari 2012; Oscar Afonso and Ana Catarina Afonso 2015; Malin Song and Shuhong Wang 2015). Thirdly, instead of
equal weights adopted in this paper and other related studies, the government may assign different weights to the consumer surplus, the producer surplus and environmental pollution. For example, the developmental state (see e.g., Carlos Aguiar de Medeiros 2011) may attach much more importance to the producer surplus rather than the consumer surplus and environmental pollution. Fourthly, this paper uses the applied theoretical approach, which lacks the empirical data to support the theoretical results. In the future research, we need to collect the relevant data in this direction.

5. Conclusion

From the 1980s, it is a consensus that privatization can cope with the inefficiency problem (e.g., the distorted production of the public firm, the shortage of the supply, and the chaotic price system) and improve social welfare. But since 1990s, after mass privatization was carried out in many countries, another trend of thought emerges to emphasize the negative effects of privatization on social welfare and strongly holds that privatization can only be carried out in some appropriate situations and implemented to a proper degree to induce higher social welfare. So in this paper we choose environmental pollution as a perspective to reflect the negative effect of privatization, and analyze when the government should privatize the public firm or not. In order to find the proper opportunity to privatize the public firm, we introduce the residents’ environmental preference to the objective function of the government. Thus, the government’s decision of privatization will be affected by the residents’ preference.

This paper finds that the residents’ environmental preference indeed plays a crucial role in the decision of privatization and at the same time it generates important effects on the results of privatization. The role and the effects are also affected by the degree of substitution, but the order of the firms’ moves will not change the main results. In sum, this paper tries to find a new mechanism for privatization and present the role of this mechanism from a greatly different perspective.

In the end, we put forward some policy implications on the basis of our theoretical analyses. Firstly, the government should take the residents’ preference (e.g., the environmental preference) into account in the privatization of the public firm. Different preferences may bring about different outcomes. Secondly, the government should consider the competition modes (e.g., Cournot competition or Stackelberg competition) of incumbent firms when it makes the decision of privatization. The competition modes play a role in affecting social welfare. Thirdly, in the process of privatization, the government should take the differentiation of products into consideration. In the sense of this paper, privatization has its own microeconomic foundations that should not be intentionally or unintentionally overlooked.
References


