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# The Role of Destructive Mechanisms within Economic Evolution

**Summary:** This research is inspired by Joseph Schumpeter's understanding of economic evolution. In his view, innovations promote economic development, whereas imitations promote the diffusion of innovations, leading the economy through a process that he defines as "creative destruction". A host of economists tend to agree on the importance and consequences of innovations and imitations within economic processes; however, opinions regarding creative destruction tend to differ. One view purports that creative destruction serves as a main variable, pushing the capitalist economic system toward equilibrium through imitation processes. A contrary view suggests that an equilibrium state actually promotes economic growth. Within this context, our research aims to model some mechanisms that may appear within economic evolution. Hurwicz's concept of economic mechanisms is introduced in a modified Arrow-Debreu model, as a way of examining Schumpeter's ideas on the role of creative destruction in economic processes that does not decrease the positions of agents. In relation to this, the present work suggests that it is indeed possible to design a mechanism that would transform the economic system under consideration toward a state of equilibrium, without making the positions of any agents worse off.

**Keywords:** Economic evolution, Innovation, Mechanism, Destruction, Equilibrium.

**JEL:** D41, L20, O12.

Determining the rules that govern economic life is at the core of interest of many economists, as shown in the studies by Uwe Cantner (2016), Richard R. Nelson (2016), Beata Ciałowicz and Andrzej Malawski (2017), Oded Stark, Fryderyk Falniowski, and Marcin Jakubek (2017), Claudio Roberto Amitrano and Lucas Vasconcelos (2019) and Stanisław Wanat, Monika Papież, and Sławomir Śmiech (2019). In these works, special attention is given to the roots, structures, and results of the economic processes presented.

The present study examines the premises of Joseph A. Schumpeter (1912, 1942) on the role of destruction in economic processes. Specifically, the aim of this research is to analyze destruction and creative destruction, by incorporating Hurwicz mechanisms (Leonid Hurwicz 1987; Todd R. Kaplan and David Wettstein 2015) in a suitably modified Arrow-Debreu model (Kenneth J. Arrow and Gerard Debreu 1954; Debreu 1959; Andreu Mas-Colell, Michael D. Whinston, and Jerry R. Green 1995; Agnieszka Lipieta 2017) as an allocation mechanism leading to equilibrium in the economy.

The Hurwicz mechanism is a mathematical structure due to which an economic activity or an institution can be formalized (Hurwicz 1987). It consists of:

- The set of all feasible information sent, consciously or unconsciously, by economic agents, called the message space; the message space is determined by the market activities of agents;
- The message correspondence, which to economic agents represented by the so-called economic environments assigns the signals (information) identified and analyzed by other agents; and
- The outcome function, which every message links with the outcome of activities of economic agents undertaken as a result of this message.

The method of determining the Hurwicz mechanism relies on the specification of its components. However, the methods used in the main part of the research are reduced to the axiomatic method and the analysis of properties of linear and continuous mappings in finite dimensional real spaces (Elliot Ward Cheney Jr. 1966). The axiomatic method seems respective, natural, and useful, especially in theoretical economics. The use of continuous mappings enables, after some identification, solving the problem of the existence of maximal or minimal values of functions essential from the point of view of economic agents.

The present research consists of seven parts. Section 1 presents the literature review, Section 2 provides an analysis of some empirical premises on the nature of destruction of economic processes, and Section 3 discusses a model of the economy and the concept of the Hurwicz mechanism. Section 4 is devoted to technical lemmas, whereas Section 5 presents some examples of modeling destruction and creative destruction mechanisms that may appear within the evolution. Finally, Section 6 provides the conclusions.

## 1. Literature Survey

The original vision of economic evolution determined by innovation was first presented by Schumpeter in the first edition of his book *Die Theorie der Wirtschaftlichen Entwicklung* (1912). In this book, the author identified essential innovative changes that could disturb the equilibrium in the economy, as well as two fundamental forms of economic life: circular flow and economic development. In the book *Capitalism, Socialism, Democracy* (1942), Schumpeter defined a mechanism clarifying the structure of the process of economic evolution, which he called creative destruction. This concept referred to the coexistence of two opposite processes: innovations resulting in the introduction of new commodities, new technologies, and new organizational structures, among others; and the processes of elimination of existing, outdated solutions. In this light, the state of equilibrium that had earlier been the aim of economic processes became the initial point of further development of the economy. In the above books, the economic mechanism, understood as the set of rules and regularities explaining the social and economic life, played a significant role (Lipieta and Malawski 2016, 2021). Schumpeter indicated two different mechanisms governing the two mentioned forms of the economy, namely, the *tatonnement* mechanism, which moves the economic system toward a state of Walras equilibrium (Leon Walras 1954), and creative destruction, which moves the economic system, through imitation processes, toward a new equilibrium state.

Schumpeter's ideas gained many followers. In 1982, the book *An Evolutionary Theory of Economic Change* by Nelson and Sidney G. Winter (1982) was published. That book initiated the neo-Schumpeterian research program and significantly developed Schumpeterian ideas, among others, by respecting the paradigm of bounded rationality (see, for instance, Friedrich A. Hayek 1945; Herbert A. Simon 1947, 1957; Armen A. Alchian 1950) and criticism of the principles of perfect rationality, as these were not reflected in the economic life. In contrast to the neoclassical and Keynesian concepts, Nelson and Winter (1982) focused on the mesosphere of the economy because this was the area of occurrence of innovative processes. It is worth emphasizing that, differently from Schumpeter, these authors applied the strict methodology of mathematical modeling of economic development.

In 1992, Philippe Aghion and Peter Howitt published a report entitled "A Model of Growth through Creative Destruction", which initiated the theory of endogenous economic growth. In that work, the authors saw the source of economic development in the effectiveness of activities of the R&D sector, which through the mechanism of creative destruction, here understood as producing commodities of higher quality, generated economic growth (see also Philippe Aghion and Peter Howitt 1998) but not in the accumulation of capital, as in the case of Solow's neoclassical theory of economic growth (see, for instance, David Romer 2012).

Innovation is essential in economic evolution. However, imitation also plays an important role in economic development as a key factor in the process of diffusion of innovation (Schumpeter 1912; Toshihiko Mukoyama 2003; James E. Bessen and Eric S. Maskin 2009; Oded Shenkar 2010; Carsten Herrmann-Pillath 2013; Nguyen H. Phus 2015; Ciałowicz and Malawski 2016).

The existence of a mechanism of transition of the economy from one form to another was justified, among others, by Horst Hanusch and Andreas Pyka (2007) and Esben S. Andersen (2009). Hanusch and Pyka (2007) explained the phenomenon of economic transition by the appearance of qualitative competition, whereas Andersen (2009) maintained that the so-called "capitalist engine", also mentioned in Schumpeter's works, caused the transitions of the economy. Moreover, Andersen identified two opposite evolutionary mechanisms. The first is the mechanism of innovation, which moves the economic system from a stationary equilibrium state to a maximally disequilibrated form, namely, the state in which "the biggest" innovative changes are observed; in every later state within the analyzed period, these changes are smaller or not noticeable. The second is the mechanism of adaptation, which moves the system back to a new stationary state, in which previous innovations have been absorbed in an equilibrated system of economic routines. Further, Yuichi Shionoya (2007) indicated the existence of mechanisms adapting innovations and directing the economic system toward equilibrium.

In the present work, the terminology used in mechanism design theory is applied to model mechanisms of economic evolution, which are formalized as Hurwicz mechanisms. The aim of mechanism design theory, proposed by Leonid Hurwicz (Hurwicz 1960), is a formal treatment of institutions and economic processes to examine how they can achieve optimal outcomes under perfect or bounded rationality, with full or partial access to knowledge (Hurwicz 1987). The concept of dispersion of private

information among economic agents led to the creation of the incentives problem (Hurwicz 1972) in the mechanism design. An approach that incorporated uncertainty into the mechanism design was initiated by Marschak and Radner (Jacob Marschak and Roy Radner 1971; Radner 1972). Kenneth Mount and Stanley Reiter (1974) analyzed mechanisms within which agents could send signals based on their private information. Later, economic mechanisms were broadly examined by Maskin (Maskin and John Riley 1984; Mathias Dewatripont and Maskin 1995; Maskin, Yingyi Qian, and Chenggang Xu 2000; Bessen and Maskin 2009) and Roger B. Myerson (1979, 1983, 1984). At present, mechanism design theory is developed in various directions and applied to several fields, such as health care, kidney exchange, and school selection, among others. Interesting results have been reported in previous works (see, for example, Alvin E. Roth, Tayfun Sönmez, and M. Utku Ünver 2004; Marek Pycia 2012; Atila Abdulkadiroğlu and Tayfun Sönmez 2013; Pycia and Ünver 2015, 2017; Abdulkadiroğlu, Parag A. Pathak, and Christopher R. Walters 2018; Tommy Andersson and Lars Ehlers 2019).

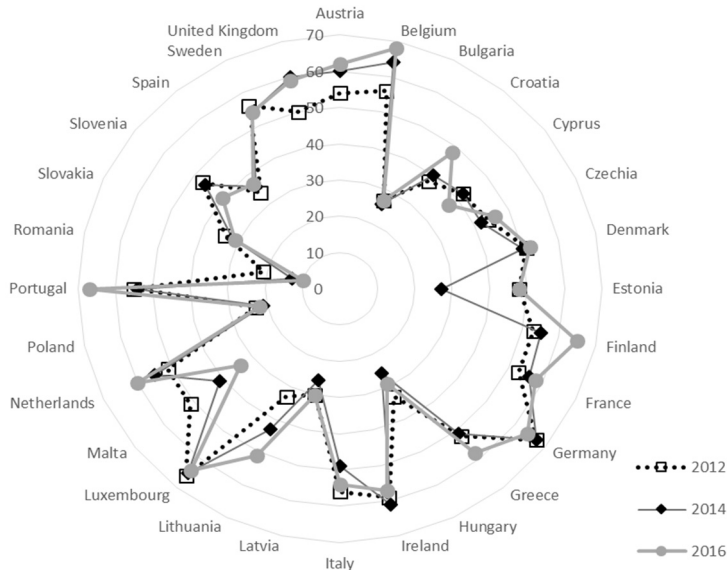
## 2. Empirical Premises on the Destructive Nature of Economic Processes

According to Schumpeter, failures of firms, commodities markets, and organizational structures, among others, are, in most cases, the result of creative destruction (Schumpeter 1912). This means that the processes of elimination of outdated commodities, technologies, economic structures, and methods of production or management are caused by the emergence of innovations, i.e., new commodities, technologies, economic structures, and methods of production or management (Schumpeter 1912). Hence, it follows from Schumpeter's theory that the market activities of innovative enterprises lead to the death as well as the birth of some enterprises.

Figures 1 to 3 show the enterprise innovating rates, enterprise death rates, and enterprise birth rates, respectively, for 28 EU countries in the years 2012, 2014, and 2016. An innovating enterprise is "an enterprise that has introduced new or improved products or services on the market or new or improved processes" (Eurostat 2019a)<sup>1</sup>, whereas the enterprise innovating rate with respect to a given reference period (usually one calendar year) is the percentage share of innovating enterprises in the total number of enterprises.

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<sup>1</sup> Eurostat. 2019a. Glossary: Innovating Enterprise. [https://ec.europa.eu/eurostat/statistics-explained/index.php/Glossary:Innovating\\_enterprise](https://ec.europa.eu/eurostat/statistics-explained/index.php/Glossary:Innovating_enterprise) (accessed November 16, 2019).



Source: Authors' work based on data from European Commission (2019)<sup>2</sup>.

**Figure 1** Enterprise Innovating Rate in EU Countries in the Years 2012, 2014 and 2016

Enterprise death refers to “the termination of an enterprise, amounting to the dissolution of a combination of production factors with this restriction that no other enterprises are involved in the event”. The enterprise death rate with respect to a given reference period (usually one calendar year) is “the number of enterprise deaths as a percentage of the population of active enterprises” (Eurostat 2019b)<sup>3</sup>. Figure 2 shows the enterprise death rates in EU countries in the years 2012, 2014, and 2016.

An enterprise is said to be born when it “starts from scratch and begins operations, amounting to the creation of a combination of production factors with the restriction that no other enterprises are involved in the event. An enterprise birth occurs when new production factors, in particular new jobs, are created”. The enterprise birth rate with respect to a given reference period (usually one calendar year) is “the number of births as a percentage of the population of active enterprises” (Eurostat 2019d)<sup>4</sup>. Figure 3 presents a comparison of enterprise birth rates in EU countries in the years 2012, 2014, and 2016.

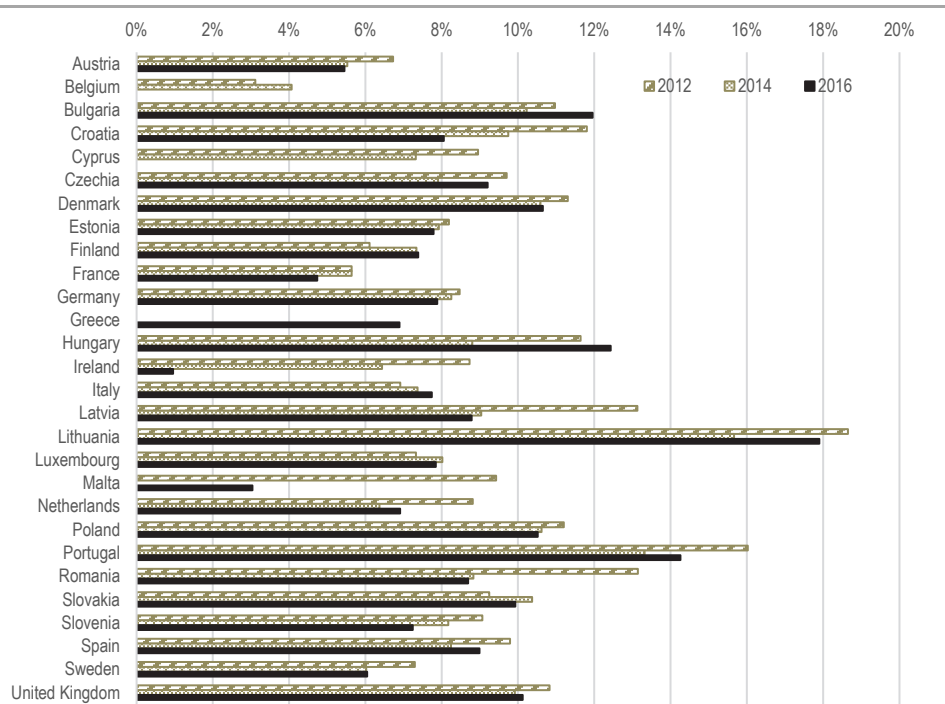
<sup>2</sup> **European Commission.** 2019. Policy Support Facility.

<https://rio.jrc.ec.europa.eu/en/stats/innovative-enterprises-total-enterprises-size-class-and-type-innovation> (accessed August 09, 2019).

<sup>3</sup> **Eurostat.** 2019b. [https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Glossary:Enterprise\\_death](https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Glossary:Enterprise_death) (accessed November 12, 2019).

<sup>4</sup> **Eurostat.** 2019d. Glossary: Enterprise Birth.

[https://ec.europa.eu/eurostat/statistics-explained/index.php/Glossary:Enterprise\\_birth](https://ec.europa.eu/eurostat/statistics-explained/index.php/Glossary:Enterprise_birth) (accessed November 12, 2019).



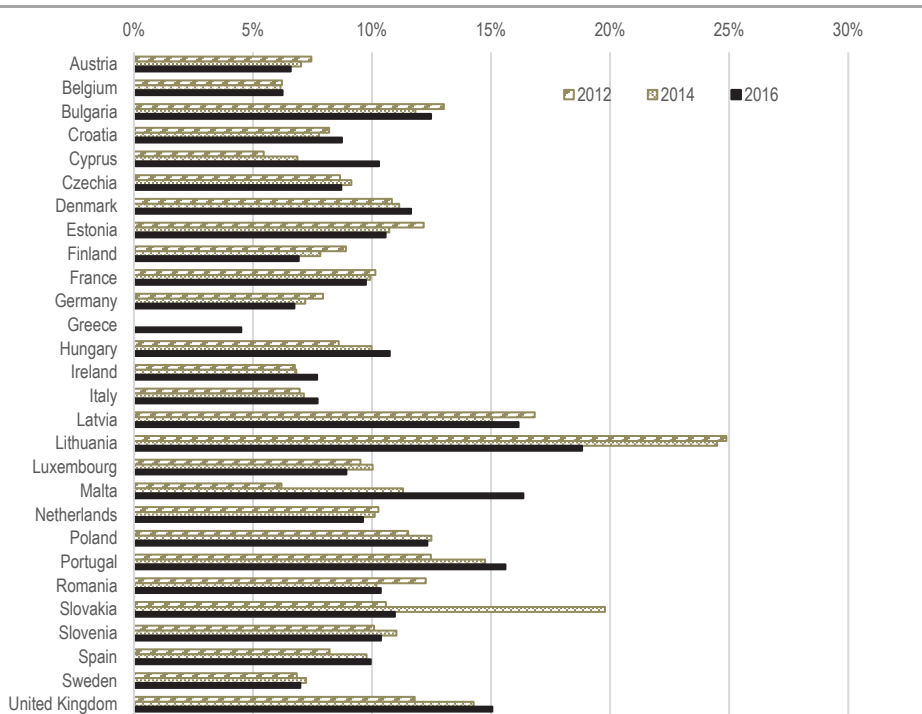
Notes: Belgium 2016, Greece 2012 and 2016, Malta 2014 - lack of data.

Source: Authors' own calculations based on data from Eurostat (2019c)<sup>5</sup>.

**Figure 2** Enterprise Death Rate in EU Countries in the Years 2012, 2014 and 2016

During the analyzed years (2012, 2014, and 2016), innovative activities of firms, as well as deaths and births of enterprises can be noted. The data seem to confirm some of Schumpeter's ideas on the effects of creative destruction. The correlation coefficient between two pairs of variables is calculated: between the enterprise innovating rate and the enterprise death rate, as well as between the enterprise innovating rate and the enterprise birth rate. The values of the correlation coefficient between the enterprise innovating rate and the enterprise death rate in the years 2012, 2014, and 2016 are -0.544, -0.354, and -0.179, respectively. The values of the correlation coefficient between the enterprise innovating rate and the enterprise birth rate in the years 2012, 2014, and 2016 are -0.454, -0.320, and -0.359, respectively. The calculated values of the correlation coefficient indicate that, in the analyzed years, an increase in the enterprise innovating rate resulted in a decrease in both the enterprise death rate and the enterprise birth rate. Additionally, the above analyzed dependencies are not strong.

<sup>5</sup> Eurostat. 2019c. Your Key to European Statistics. <https://ec.europa.eu/eurostat/tgm/refreshTableAction.do?tab=table&plugin=1&pcode=tin00170&language=en> (accessed November 12, 2019).



Source: Authors' own calculations based on data from Eurostat (2019d).

**Figure 3** Enterprise Birth Rate in EU Countries in the Years 2012, 2014 and 2016

To further analyze the results of creative destruction, destructive mechanisms of economic evolution are modeled. Considering both Schumpeter's theory and the data analysis, it is assumed that within the economic evolution, new commodities, technologies, and enterprises, among others, can emerge on the market and outdated ones can be eliminated from the market.

### 3. The Model

The analysis of mechanisms in the approach presented in this work was proposed by Lipieta (2015), whereas the model of the evolution of the economy presented below was defined in Lipieta and Artur Lipieta (2017) and explored in Lipieta and Malawski (2021).

The number of economic agents operating in the economy, as well as the number of commodities produced and consumed in the economy, are finite. However, in the model presented below, the number of producers and consumers and the number of commodities are determined as countable. Such an approach is justified because an almost countable number of economic agents is assumed to be inactive, and an unknown number of new commodities can be produced in the future. An inactive agent at a given moment is an agent whose activity is reduced to the zero plan at that moment.

Every inactive agent can be interpreted as a potential future agent who is waiting for the proper time to enter the market.

Time is considered as a discrete *variable*. Suppose  $t_0, t_1 \in \{0, 1, 2, \dots\}$ ,  $t_0 < t_1$ , are separate time periods within which there are no changes in agent activities,  $t \in \{t_0, t_1\}$ . Without loss of generality, we assume that  $t_0 = 0$  and  $t_1 = 1$ . Let:

- $A = \{a_i\}_{i \in \mathbb{N}}$  be a countable set of consumers and
- $B = \{b_j\}_{j \in \mathbb{N}}$  be a countable set of producers.

Under the previous arrangements, there exist numbers  $m_t, n_t \in \{1, 2, \dots\}$  such that, for every  $i > m_t$  and for every  $j > n_t$ , every consumer  $a_i$  and every producer  $b_j$  are inactive.

Let  $\ell_t \in \mathbb{N}_+ \stackrel{\text{def}}{=} \{1, 2, \dots\}$  be the number of the commodities that are produced and consumed in the economy at period  $t$  or which were previously produced and consumed. Hence,  $\ell_0 \leq \ell_1$ . We define:

$$\mathcal{R}^{\ell_t} \stackrel{\text{def}}{=} \mathbb{R}^{\ell_t} \times \{0\} \times \{0\} \times \dots \subset \mathcal{R}, \text{ where } \mathcal{R} \stackrel{\text{def}}{=} \mathbb{R} \times \mathbb{R} \times \dots$$

and a scalar product of vectors  $x, y \in \mathcal{R}^{\ell_t}$  by the standard rule:  $x \circ y = \sum_{l \in \mathbb{N}} x_l y_l$ .

Space  $\mathcal{R}^{\ell_t}$  is interpreted as the commodity-price space in the analyzed economy at period  $t$ , whereas every coordinate  $l \in \{\ell_t + 1, \ell_t + 2, \dots\}$  is interpreted as the quantity of future goods. Hence,  $\mathcal{R}^{\ell_0} \subset \mathcal{R}^{\ell_1}$ . Such an approach simplifies the description of the processes in which the set of commodities and the number of economic agents can be changed in time.

The characteristics of economic agents are defined below. A production activity of producer  $b$  at period  $t$ , feasible with respect to technologies, is identified with vector  $y^b(t) \in \mathcal{R}^{\ell_t}$ , called producer's  $b$  production plan. All production plans of producer  $b$  at period  $t$  form the so-called production set  $Y^b(t) \subset \mathcal{R}^{\ell_t}$ . By the previous arrangements:

$$\forall j > n_t \ Y^{b_j}(t) \stackrel{\text{def}}{=} \{0\}$$

and there is a producer  $b \in B$ , for whom at least one production plan  $y^b(t) \in Y^b(t)$  has an  $\ell_t$ -th coordinate different from zero. The set of active producers at time  $t$  is noted by  $B_t$  ( $B_t = \{1, \dots, n_t\}$ ). Similarly, let:

- $X^a(t) \subset \mathcal{R}^{\ell_t}$  be the consumption set of consumer  $a$ , where:

$$\exists m_t \in \{1, 2, \dots\} \ \forall i > m_t \ X^{a_i}(t) \stackrel{\text{def}}{=} \{0\},$$

which means that, for  $i > m_t$ , every consumer  $a_i$  is inactive; the set of active consumers at time  $t$  is noted by  $A_t$  ( $A_t = \{1, \dots, m_t\}$ ). Let:

- $\Xi_t$  be the set of preference relations in space  $\mathcal{R}^{\ell_t}$ ;
- $\preceq_t^a \in X^a(t) \times X^a(t)$  be the preference relation of consumer  $a$ ;
- $\omega^a(t) \in X^a(t)$  be the initial endowment of consumer  $a$ ;
- $\omega(t) = \sum_{a \in A} \omega^a(t) \in \mathcal{R}^{\ell_t}$  be the total endowment of the economy; and
- function  $\theta_t: A \times B \rightarrow [0, 1]$ , where



- $\theta_t(a, b) = 0$  if  $a \in A \setminus A_t$  or  $b \in B \setminus B_t$ ;
- number  $\theta_t(a, b) \in [0,1]$ , for every  $a \in A_t$  and  $b \in B_t$ , is the share of consumer  $a$  in the profit of producer  $b$ ; and
- $\sum_{a \in A_t} \theta_t(a, b) = 1$ , for every  $b \in B_t$

is the share function.

Let  $K \stackrel{\text{def}}{=} A \cup B$ . By the previous arrangements  $K = \{k_1, k_2, \dots\}$ , there exists  $\kappa \in \{1, 2, \dots\}$ ,  $\kappa \leq m_0 + n_0$ , such that every agent  $k_r$ , for  $r > \kappa$ , is inactive as a producer and consumer. Based on the above notation, the environment  $e^k(t)$  of every economic agent  $k \in K$  (Arrow and Michael D. Intriligator 1987) at period  $t$  is defined. That is:

$$e^k(t) = (Y^k(t), X^k(t), \omega^k(t), \tilde{\varepsilon}_t(k), \tilde{\theta}_t(k, \cdot)), \tag{1}$$

where:

- $Y^k(t) = \{0\}$  for  $k \notin B$ ,
- $X^k(t) = \{0\}$  for  $k \notin A$ ,
- $\omega^k(t) = 0$  for  $k \notin A$ ,
- $\tilde{\varepsilon}_t(k) = \leq_t^a$  for  $k \in A$ ,  $\tilde{\varepsilon}_t(k) = \{\emptyset\}$  for  $k \notin A$ , and
- mapping  $\tilde{\theta}_t: K \times K \rightarrow [0,1]$  is the extension of mapping  $\theta_t$  in set  $K \times K$  such that  $\tilde{\theta}_t(k, \cdot) \equiv 0$  for  $k \notin A$ ,  $\tilde{\theta}_t(\cdot, k) \equiv 0$  for  $k \notin B$ ,  $\tilde{\theta}_t(a, b) = \theta_t(a, b)$  for  $a \in A$  and  $b \in B$ .

By the above,  $e^k(t) \in E^k(t)$ , with  $\mathcal{F}(K, [0,1]) \stackrel{\text{def}}{=} \{f \mid f: K \rightarrow [0,1]\}$ . Set  $E^k(t)$  is the set of all feasible environments of agent  $k$  at period  $t$ . Set:

$$E(t) \stackrel{\text{def}}{=} E^{k_1}(t) \times E^{k_2}(t) \times \dots \tag{2}$$

is the set of environments at period  $t$ . Vector:

$$e(t) = (e^{k_1}(t), e^{k_2}(t), \dots) \in E(t) \tag{3}$$

is the environment at period  $t$ . The components of environment  $e^k(t)$  are not changed at period  $t$ .

Note that the components of environment  $e(t)$  (see (1)) form a private ownership economy, denoted below by  $\mathcal{E}(t)$ , with space  $\mathcal{R}^{\ell t}$  as the commodity-price space (compare with Debreu 1959; Mas-Colell, Whinston, and Green 1995; Lipieta 2017). Recall that if in economy  $\mathcal{E}(t)$  there exists a sequence:

$$(x^*(t), y^*(t), p(t)),$$

where  $x^*(t) = (x^{k_1^*}(t), x^{k_2^*}(t), \dots)$ ,  $y^*(t) = (y^{k_1^*}(t), y^{k_2^*}(t), \dots)$ ,  $p(t) \in \mathcal{R}^{\ell t}$  such that:

- $y^{k^*}(t)$  maximizes the profit of producer  $k$  at price vector  $p(t)$  in set  $Y^k(t)$ , if  $k \in B$ ;  $y^{k^*}(t) = 0$ , if  $k \notin B$ ;
- $x^{k^*}(t)$  maximizes the preferences of consumer  $k$  in a nonempty budget set

$$\beta_t^a(p(t)) \stackrel{\text{def}}{=} \left\{ x^a(t) \in X^a(t) : p(t) \circ x^a(t) \leq p \circ \omega^a(t) + \sum_{b \in B_t} \theta_t(a, b) \cdot p(t) \circ y^{k^*}(t) \right\}, \tag{4}$$

if  $k \in A$ , as well as  $x^{k^*}(t) = 0$ , if  $k \notin A$ ; and

$$\blacksquare \sum_{k \in K} x^{k*}(t) - \sum_{k \in K} y^{k*}(t) = \omega(t),$$

then there is a state of equilibrium in economy  $\mathcal{E}(t)$  (Arrow and Debreu 1954; Mas-Colell, Whinston, and Green 1995). If economy  $\mathcal{E}(t)$  is in equilibrium, then the economic agents realize their plans of action, which, at prices  $p(t)$ , form a state of equilibrium. Hence, economy  $\mathcal{E}(t)$  in equilibrium is in the form of a circular flow (Schumpeter 1912; Lipieta and Malawski 2021).

The sequence:

$$m^k(t) \stackrel{\text{def}}{=} (p(t), \check{y}^k(t), \check{x}^k(t)), \quad (5)$$

where:

- $\check{x}^k(t) \in X^k(t)$  is a plan of action of consumer  $k \in K$  at period  $t$  and
- $\check{y}^k(t) \in Y^k(t)$  is the producer's  $k \in K$  plan of action at period  $t$  is interpreted as a message of agent  $k \in K$  at period  $t$ . The set of all feasible messages of the form (5), denoted by  $M^k(t)$ , is contained in set  $\mathcal{R}^{\ell_t} \times \mathcal{R}^{\ell_t} \times \mathcal{R}^{\ell_t}$ . Vector:

$$m(t) \stackrel{\text{def}}{=} (m^{k_1}(t), m^{k_2}(t), \dots) \in M^{k_1}(t) \times M^{k_2}(t) \times \dots \quad (6)$$

is the message at period  $t$ . Suppose that:

$$M(t) \subset M^{k_1}(t) \times M^{k_2}(t) \times \dots \text{ and } M(t) \neq \emptyset.$$

Now recall the definition of the economic mechanism in the Hurwicz sense.

**Definition 1.** (Compare with Hurwicz 1987; Lipieta and Malawski 2021). The triple  $\Gamma_t = (M(t), \mu_t, h_t)$ , where:

- $\mu_t: E(t) \rightarrow M(t)$  is the message correspondence and
- $h_t: M(t) \rightarrow Z$  is the outcome function,

is the mechanism in the sense of Hurwicz or the Hurwicz mechanism.

Outcome function  $h_t$  to every message  $m(t) \in M(t)$  (see (5) and (6)) assigns an allocation, which is the result of the retrieval and analysis of message  $m(t)$  by economic agents. Message correspondence  $\mu_t$  to every environment  $e(t)$  assigns the set of messages, consciously or unconsciously sent at period  $t$  by economic agents.

Sequence  $P(t) = (B, \mathcal{R}^{\ell_t}, (Y^k(t))_{b \in B}, p(t))$  is the mathematical equivalent of the production sphere of an economy at period  $t$ . Sequence  $P(t)$  is the production system of economy  $\mathcal{E}(t)$ . Under the previous arrangements, system  $P(1)$  is the transformation of system  $P(0)$ , economy  $\mathcal{E}(1)$  - the transformation of economy  $\mathcal{E}(0)$ .

**Definition 2.** Production system  $P(1)$  is the imitative transformation of production system  $P(0)$ ; i.e.,  $P(0) \subset_{imt} P(1)$ , if:

1.  $\ell_0 = \ell_1$  and
2.  $\forall b \in B Y^b(1) \subset \cup_{b \in B} Y^b(0)$ .

If  $P(0) \subset_{imt} P(1)$  and, additionally,

3.  $\forall b \in B Y^b(0) \subset Y^b(1)$  and
4.  $\forall b \in B \forall y^b(0) \in Y^b(0) \exists y^b(1) \in Y^b(1): p(0) \circ y^b(0) \leq p(1) \circ y^b(1)$ ,

then production system  $P(1)$  is the cumulative transformation of production system  $P(0)$ ; i.e.,  $P(0) \subset_{ct} P(1)$ .

In the imitative transformation  $P(1)$  of production system  $P(0)$ , there are neither new commodities nor technologies (conditions 1 and 2). If  $P(1)$  is the cumulative transformation of production system  $P(0)$ , then, additionally, firms are not eliminated from the market (condition 3), the economic positions of producers (Lipieta and Malawski 2016) are not worse than in the initial system, meaning that adequate profits in system  $P(1)$  are not less than in system  $P(0)$  (condition 4). If there are no changes in the economy between periods  $t = 0$  and  $t = 1$ ,  $P(0) \subset_{ct} P(1)$ , and the characteristics of consumers are constant at interval  $[0,1]$ , then the economy is in the form of a circular flow in periods  $t = 0$  and  $t = 1$  and in the period between them.

**Definition 3.** Production system  $P(1)$  is the innovative transformation of production system  $P(0)$ ; i.e.,  $P(0) \subset_{it} P(1)$ , if:

- 1)  $\ell_0 = \ell_1 \Rightarrow \exists b_{in} \in B \exists y^{b_{in}}(1) \in Y^{b_{in}}(1): y^{b_{in}}(1) \notin \cup_{b \in B} Y^b(0)$  and
- 2)  $\ell_0 < \ell_1 \Rightarrow \exists b_{in} \in B \exists y^{b_{in}}(1) \in Y^{b_{in}}(1): y^{b_{in}}(1) \notin \cup_{b \in B} (Y^b(0) \times \{0\} \times \dots) \subset \mathcal{R}^{\ell_1}$ .

Note that if  $\ell_0 = \ell_1$  and  $P(0) \subset_{it} P(1)$ , then new technologies are the only innovations at period  $t = 1$  with respect to period  $t = 0$  (condition 1). If  $\ell_0 < \ell_1$  and  $P(0) \subset_{it} P(1)$ , then a new commodity is introduced and every innovator introduces a new technology into the production sphere at period  $t = 1$  with respect to period  $t = 0$  (condition 2). Producer  $b_{in}$  satisfying conditions 1 or 2 by Definition 3 is an innovator, and vectors  $y^{b_{in}}(1)$  are his innovative plans. It is assumed that, if  $P(0) \subset_{it} P(1)$ , then at period  $t = 1$  at least one innovator realizes one of his innovative plans, which is coherent with Schumpeter's theory. If innovations are introduced, then the economy is not in the form of a circular flow, and the economic development has already started. The producer, who is not the innovator, is called the imitator.

Consider private ownership economies  $\mathcal{E}(0)$  and  $\mathcal{E}(1)$ , by the use of which producer and consumer activities on markets are modeled at two subsequent moments of time. Let  $\Gamma_0$  be a mechanism whose environments at period  $t = 0$  form economy  $\mathcal{E}(0)$  and whose outcomes are observed in economy  $\mathcal{E}(1)$ . Now the following definitions can be formulated:

**Definition 4.** Mechanism  $\Gamma_0$  is innovative, if  $P(0) \subset_{it} P(1)$ . Mechanism  $\Gamma_0$  is imitative, if  $P(0) \subset_{imt} P(1)$ .

## 4. Technical Lemmas

Let private ownership economy  $\mathcal{E}(0)$  be a mathematical equivalent of an economy at period  $t = 0$ . Let  $p \in \mathcal{R}^{\ell_0}$  be a price system that can be but does not have to be the market price vector at time  $t = 0$ , i.e.,  $p(0) = p$  or  $p(0) \neq p$ . Suppose that there exists an allocation:

$$\left( (x^{k^*}(0))_{k \in K'}, (y^{k^*}(0))_{k \in K} \right)$$

such that at price system  $p$ ,

- $y^{k*}(0)$  maximizes the profit of producer  $k$  in set  $Y^k(0)$ , if  $k \in B$ ;
- $y^{k*}(0) = 0$ , if  $k \notin B$ ;
- $x^{k*}(0)$  maximizes the preferences of consumer  $k$  in a non-empty set

$\beta^a(p) = \{x^a(0) \in X^a(0) : p \circ x^a(0) \leq p \circ \omega^a(0) + \sum_{b \in B} \theta_0(a, b) \cdot p \circ y^{k*}(0)\}$ ,  
if  $k \in A$ ; and

- $x^{k*}(0) = 0$ , if  $k \notin A$ .

Note that if  $p(0) = p$ , then  $\beta^a(p) = \beta_0^a(p(0))$  for every  $a \in A$  (see (4)). Let

$$\zeta \stackrel{\text{def}}{=} \sum_{k \in K} x^{k*}(0) - \sum_{k \in K} y^{k*}(0) - \sum_{k \in K} \omega^k(0). \tag{7}$$

Recall that  $B_0 = \{b_1, \dots, b_{n_0}\}$  denotes the set of active producers in economy  $\mathcal{E}(0)$ . Under the above arrangements, the following is true:

**Lemma 1.** Suppose that  $\zeta = 0$  and that there exists a linear proper subspace  $V$  of space  $\mathcal{R}^{\ell_0}$  such that:

$$\forall a \in A \quad X^a(0) \subset V. \tag{8}$$

Then there exists a continuous mapping  $\Phi: \mathcal{R}^{\ell_0} \rightarrow \mathcal{R}^{\ell_0}$  such that there is equilibrium in economy  $\mathcal{E}(1)$ , in which the conditions:

$$Y^k(1) = \Phi(Y^k(0)) \text{ for every } k \in K \tag{9}$$

and

$$\left\{ \begin{array}{ll} Y^k(1) \subset V & \text{for } k \in B_0 \\ Y^k(1) = Y^k(0) \subset V & \text{for } k \in K \setminus B_0 \\ X^k(1) = X^k(0), \leq_1^k = \leq_0^k, \omega^k(1) = \omega^k(0) & \text{for } k \in K \\ \Xi_1 = \Xi_0, \theta_1 = \theta_0 & \end{array} \right. \tag{10}$$

are satisfied.

**Proof.** If  $V$  is a linear proper subspace of  $\mathcal{R}^{\ell_0}$ , then  $V|_{\mathbb{R}^{\ell_0}}$  is a linear proper subspace of  $\mathbb{R}^{\ell_0}$ . Hence, for subspace  $V$ , there exists a natural number  $d \in \{1, 2, \dots, \ell_0 - 1\}$  and linearly independent vectors  $g^1, \dots, g^d \in \mathcal{R}^{\ell_0}$ ,  $g^s = (g_1^s, \dots, g_{\ell_0}^s, 0, \dots)$  for  $s \in \{1, 2, \dots, d\}$  such that:

$$V = \bigcap_{s=1}^d \ker \tilde{g}^s, \tag{11}$$

where:

$$\tilde{g}^s: \mathcal{R}^{\ell_0} \ni (x_1, \dots, x_{\ell_0}, 0, \dots) \rightarrow g_1^s x_1 + \dots + g_{\ell_0}^s x_{\ell_0} \in \mathbb{R} \tag{12}$$

is, for every  $s \in \{1, \dots, d\}$ , a linear and continuous mapping, and  $\ker \tilde{g}^s = (\tilde{g}^s)^{-1}(0)$ . Consider vectors  $q^1, \dots, q^d \in \mathcal{R}^{\ell_0}$ , a solution of the system of equations:

$$\tilde{g}^s(q^r) = \delta^{sr} \text{ for } s, r \in \{1, \dots, d\}, \tag{13}$$

where:

$$\delta^{sr} = \begin{cases} 1 & \text{if } s = r \\ 0 & \text{if } s \neq r \end{cases} \tag{14}$$

is the Kronecker delta. Then, mapping  $Q: \mathcal{R}^{\ell_0} \rightarrow V$  of the form:

$$Q(x) = x - \sum_{s=1}^d \tilde{g}^s(x) \cdot q^s \tag{15}$$

is a linear and continuous projection on subspace  $V$  determined by vectors  $q^1, \dots, q^d$  (Cheney Jr. 1966).

If  $p \in V^T$ , then mapping  $Q$  of form (15) satisfies:

$$\forall v \in V: p \circ v = p \circ Q(v) = 0. \tag{16}$$

If  $p \notin V^T$ , then vectors  $p, g^1, \dots, g^d \in \mathcal{R}^{\ell_0}$  are linearly independent. Hence, there exists a sequence  $q^1, \dots, q^d \in \mathcal{R}^{\ell_0}$ , a solution of (13) additionally satisfying:

$$p \circ q^s = 0 \quad \text{for } s \in \{1, \dots, d\}. \tag{17}$$

It is not difficult to verify that if  $p \notin V^T$ , then projection  $Q$  of form (15) determined by vectors  $q^1, \dots, q^d$  calculated by the system of conditions (13) and (17) satisfies:

$$\forall y \in \mathcal{R}^{\ell_0}: p \circ y = p \circ Q(y). \tag{18}$$

The rationale is the same as that in the proof of Theorem 3.1 in Lipieta (2010).

A) If  $p \in V^T$ , then by (16), every projection  $Q$  of form (15) determined by any vectors  $q^1, \dots, q^d \in \mathcal{R}^{\ell_0}$  calculated by (13) indicates a state of equilibrium in economy  $\mathcal{E}(1)$ , in which condition (10) is satisfied, i.e., sequence:

$$((x^{k^*}(1))_{k \in K}, (y^{k^*}(1))_{k \in K}, p(1)), \tag{19}$$

where:

$$x^{k^*}(1) = x^{k^*}(0), y^{k^*}(1) = Q(y^{k^*}(0)) \text{ for } k \in K, p(1) = p. \tag{20}$$

Hence,  $\Phi = Q$ .

B) If  $p \notin V^T$ , then property (18) of projection  $Q$  of the form (15) determined by vectors  $q^1, \dots, q^d \in \mathcal{R}^{\ell_0}$  calculated by (13) and (17) indicates that sequence (19) satisfying (20) is a state of equilibrium in such economy  $\mathcal{E}(1)$ , which is formed by components of environment  $e(1)$  (see (1) and (3)) satisfying conditions (9) and (10).

Note that if  $\ell_0 > 1$ , then the system of Equations (13) has infinitely many solutions. Hence, if  $p \in V^T$  and  $\ell_0 > 1$ , then there are infinitely many ways, i.e., mappings of the form (15), to obtain equilibrium in economy  $\mathcal{E}(1)$ . The same is valid and because of the same reasons, if  $\ell_0 > 2$  and  $p \notin V^T$ .

**Lemma 2.** Suppose that condition (8) is satisfied,  $\zeta \neq 0$  and  $p \circ \zeta = 0$ . Then there exists a continuous mapping  $\Phi: \mathcal{R}^{\ell_0} \rightarrow \mathcal{R}^{\ell_0}$  such that there is equilibrium in economy  $\mathcal{E}(1)$ , in which conditions (9) and (10) are satisfied.

**Proof.** A) Suppose that  $\zeta \notin V$ . We can choose such functionals  $\tilde{g}^1, \dots, \tilde{g}^d$  of form (12) giving (11), where  $\tilde{g}^1(\zeta) = 1$  and  $\tilde{g}^2(\zeta) = \dots = \tilde{g}^d(\zeta) = 0$  if  $d > 1$ . Let

$q^1 = \zeta$  and  $q^2, \dots, q^d \in \mathcal{R}^{\ell_0}$  be calculated from (13), if  $p \in V^T$ , or from (13) and (17), if  $p \notin V^T$ . Mapping  $Q$  of form (15) determined by vectors  $q^1, \dots, q^d$  satisfies (18). Hence, there is a state of equilibrium in economy  $\mathcal{E}(1)$  of form (19) satisfying (20), in which condition (10) is valid (compare with the proof of Theorem 4.1 in Lipieta 2015). In that case, as above,  $\Phi = Q$ .

B) Now suppose that  $\zeta \in V$ . Take any projection  $Q$  of form (15) determined by any vectors  $q^1, \dots, q^d \in \mathcal{R}^{\ell_0}$ , where  $q^1 = \zeta$  and  $q^2, \dots, q^d \in \mathcal{R}^{\ell_0}$  are calculated from (13), if  $p \in V^T$ , or from (13) and (17), if  $p \notin V^T$ . For  $y \in \mathcal{R}^{\ell_0}$ , we have:

$$\Phi(y) \stackrel{\text{def}}{=} Q(y) + \frac{1}{n_0} \cdot \zeta. \tag{21}$$

Note that  $\Phi(y) \in V$  for every  $y \in \mathcal{R}^{\ell_0}$ . By (18), vector  $y^b(1) = Q(y^{b*}(0)) + \frac{1}{n_0} \cdot \zeta$  for  $b \in B_0$  maximizes the profit in set  $Y^b(1) \stackrel{\text{def}}{=} \Phi(Y^b(0)) = Q(Y^b(0)) + \frac{1}{n_0} \cdot \zeta$ . Consequently, sequence (19), where  $x^{k*}(1) = x^{k*}(0)$  for  $k \in K$ ,  $y^{k*}(1) = \Phi(y^{k*}(0)) = Q(y^{k*}(0)) + \frac{1}{n_0} \cdot \zeta$  for  $k \in B_0$ , and  $y^{k*}(1) = y^{k*}(0)$  for  $k \in K \setminus B_0$  is the state of equilibrium in such economy  $\mathcal{E}(1)$ , which is formed by components of environment  $e(1)$  (see (1) and (3)) satisfying conditions (9) and (10).

Recall that  $A_0 = \{a_1, \dots, a_{m_0}\}$  denotes the set of active consumers at period  $t = 0$ . Now we suggest:

**Lemma 3.** If  $\zeta \neq 0$  and  $p \circ \zeta \neq 0$ , then there exists an equilibrium in economy  $\mathcal{E}(1)$ , in which:

$$\left\{ \begin{array}{ll} Y^k(1) = Y^k(0) & \text{for } k \in K \setminus \{b_{n_0+1}\} \\ Y^k(1) \neq Y^k(0) & \text{for } k = b_{n_0+1} \\ X^k(1) = X^k(0), \leq_1^k = \leq_0^k, \omega^k(1) = \omega^k(0) & \text{for } k \in K \\ \Xi_1 = \Xi_0, \theta_1 = \theta_0 & \end{array} \right. \tag{22}$$

**Proof.** Because, for every  $a \in A$ ,

$$p \circ x^{a*}(0) \leq p \circ \omega^a(0) + \sum_{b \in B} \theta_0(a, b) \cdot (p \circ y^{b*}(0)),$$

then  $p \circ \zeta < 0$ . Hence, for some  $a \in A$ ,

$$p \circ x^{a*}(0) < p \circ \omega^a(0) + \sum_{b \in B} \theta_0(a, b) \cdot (p \circ y^{b*}(0)). \tag{23}$$

For every consumer  $a \in A_0$  for which condition (23) is valid, there exists number  $\alpha_a > 0$  such that:

$$p \circ x^{a*}(0) = p \circ \omega^a(0) + \sum_{b \in B} \theta_0(a, b) \cdot (p \circ y^{b*}(0)) + \alpha_a \cdot (p \circ \zeta).$$

For every  $a \in A_0$ , for which:

$$p \circ x^{a*}(0) = p \circ \omega^a(0) + \sum_{b \in B} \theta_0(a, b) \cdot (p \circ y^{b*}(0)),$$

we have  $\alpha_a = 0$ .

Consequently, by the fact that  $p \circ \zeta < 0$ , there exists a sequence of nonnegative numbers  $(\alpha_a)_{a \in A}$  such that  $\sum_{a \in A} \alpha_a = 1$ ,  $\alpha_a = 0$  for  $a \notin A_0$ . Let  $Y^{b_{n_0+1}}$  be a non-empty set satisfying:

$$Y^{b_{n_0+1}} \subset W + \{\zeta\} \text{ and } \zeta \in Y^{b_{n_0+1}} \quad (24)$$

for  $W = \{y \in \mathcal{R}^{\ell_0} : p \circ y \leq 0\}$ . Then, by (24), vector  $\zeta$  maximizes the function:

$$Y^{b_{n_0+1}} \ni y \rightarrow p \circ y \in \mathbb{R}.$$

Based on the above,  $\beta_1^a(p(1)) = \beta^a(p)$  for every consumer  $a$ . It is not difficult to verify that sequence (19), in which

$$x^{k^*}(1) = x^{k^*}(0), y^{k^*}(1) = y^{k^*}(0) \text{ for } k \in K \setminus \{b_{n_0+1}\}, y^{b_{n_0+1}}(1) = \zeta, p(1) = p,$$

is a state of equilibrium in economy  $\mathcal{E}(1)$ , in which condition (22) is valid.

## 5. Destruction and Creative Destruction within Economic Evolution

Destruction within economic processes is apparent, among others, by the elimination of existing products, technologies, or firms, as well as if the economic position of at least one agent is worse than it was earlier. If a destructive mechanism results in the elimination of a harmful commodity or technology, then it can be regarded as an eco-mechanism because it leads to environment-friendly changes. Thus, destructive mechanisms can have positive or negative outcomes, whereas failures of firms and markets, among others, lead to a disequibrated economy. The present research focused on the positive outcomes of destructive mechanisms; in particular, it analyzed the possibility of improving the position of agents in an equilibrated transformation of the initial economy.

The positions of economic agents are defined based on a comparison between producers' profits and consumers' optimal plans in two periods. If the profit of a producer is not less at present than it was at an earlier period, then the present economic position of that producer is not worse than it was at that earlier time. Similarly, if a realized consumption plan of a consumer is not worse than his consumption plan realized at an earlier period, then the present economic position of that consumer is not worse than it was at that earlier time. The details about the positions of economic agents are reported in Lipieta and Malawski (2016).

Based on the above, the following definition is proposed:

**Definition 5.** If:

$$\exists b \in B \exists y^b(0) \notin \cup_{b \in B} Y^b(1) \text{ or } \exists b \in B : (Y^b(1) = \{0\} \wedge Y^b(0) \neq \{0\}),$$

then production system  $P(1)$  is the destructive transformation of production system  $P(0)$ ; i.e.,  $P(0) \subset_{dt} P(1)$ . Mechanism  $\Gamma_0$  is destructive if  $P(0) \subset_{dt} P(1)$ .

As previously mentioned, creative destruction can be regarded as the synthesis of two opposing tendencies, namely, creative innovations and elimination of existing products and organizational structures, along with their replacement by new ones (Schumpeter 1912; Lipieta and Malawski 2016). Therefore, the following is proposed:

**Definition 6.** If  $\Gamma_0$  is a destructive and innovative mechanism, then it is called a mechanism of creative destruction.

In the spirit of Schumpeter's creative destruction principle, if consumers do not find a commodity or a set of commodities attractive, then they would not be interested in buying those commodities. Hence, the consumption plans of action, modeled in the Arrow and Debreu apparatus, are linear (see Lipieta 2015). The linearity of consumption plans means that the plans are contained in a proper linear subspace of the commodity space. Consumption plans can also be linear if consumers are not interested in buying goods produced with the application of some technologies (for example, if the commodities are harmful or dangerous or if the application of other technologies provides more functional or nicer commodities). The linearity of all consumer plans in this setup implies the linearity of consumption sets, which means that all consumption sets are also contained in the proper subspace of the commodity space. Hence, it can be assumed that, at the beginning of the analyzed process of Schumpeterian evolution, the consumption sets are linear. The latter could also take place, among others, if some technologies appear to be unacceptable, i.e., unethical or harmful. Therefore, if there exists a linear subspace  $V$  of commodity-price space  $\mathcal{R}^{\ell_0}$  such that condition (8) is valid or the total consumption plan belongs to  $V$ , then this means that every consumer or consumers as the whole, respectively, are not interested in the consumption of some commodities or do not want to consume some combinations of goods.

Consumer choices can force producers to change their technologies to meet the market demand. The modification of a production sphere aims to satisfy analogical to (8) property for production sets, namely,

$$\forall b \in B \ Y^b(0) \subset \tilde{V}, \quad (25)$$

for a linear subspace  $\tilde{V}$  of commodity-price space  $\mathcal{R}^{\ell_0}$ . Condition (25) means that the production sets are linear (James C. Moore 2007; Lipieta 2015). In some cases,  $V = \tilde{V}$ ; in others,  $V \neq \tilde{V}$ .

Let  $\mathcal{E}(0)$  be a private ownership economy. Consider a price system  $p$  that can be but does not have to be the market price system at time  $t = 0$ , i.e.,  $p(0) = p$  or  $p(0) \neq p$ . As earlier, suppose that there exists an allocation:

$$((x^{k^*}(0))_{k \in K}, (y^{k^*}(0))_{k \in K})$$

such that  $y^{k^*}(0)$  maximizes the profit of producer  $k$  at price vector  $p$  in set  $Y^k(0)$ , if  $k \in B$ ;  $y^{k^*}(0) = 0$ , if  $k \notin B$ ,  $x^{k^*}(0)$  maximizes the preferences of consumer  $k$  in set  $\beta^a(p)$ , if  $k \in A$ ,  $x^{k^*}(0) = 0$ , if  $k \notin A$ . Let  $\zeta$  be of the form (7).

Below are some mechanisms resulting in equilibrium, a number of which are shown to be mechanisms of creative destruction.

**Theorem 1.** If  $p \circ \zeta = 0$  and there exists a linear subspace  $V$  of space  $\mathcal{R}^{\ell_0}$  satisfying (8), then there exist:

- (1) An economy  $\mathcal{E}(1)$  - the transformation of economy  $\mathcal{E}(0)$ , in which conditions (9) and (10) are satisfied; and
- (2) A mechanism  $\Gamma_0$ , which results in equilibrium in economy  $\mathcal{E}(1)$ .



**Proof.** To determine an economic mechanism, the following are specified:

- the environment at time  $t = 0$  of the form (3), and every environment of agent  $k$  at time  $t$  of the form (1); consequently,  $E(0)$  of the form (2);
- the message at time  $t = 0$  of the form (6), and every message of agent  $k$  at that time of the form (5) and, additionally,  $\tilde{y}^k(0) = y^{k*}(0)$ , as well as  $\tilde{x}^k(0) = x^{k*}(0)$ ;
- $Z = \left\{ \begin{array}{l} (x^*(1), y^*(1), p(1)): \\ ((x^*(1), y^*(1), p(1)) \text{ is a state of equilibrium in economy } \mathcal{E}(1)) \end{array} \right\}$ ;
- $\mu_0: E(0) \rightarrow M(0)$ , where  $\mu(e(0)) = m(0)$ ; and
- $h_0: M(0) \rightarrow Z$ ,  $h(m) = ((x(1), y(1), p(1)))$ ,

where  $x(1) = (x^{k_1}(0), x^{k_2}(0), \dots)$ ,  $y(1) = (\Phi(y^{k_1}(0)), \Phi(y^{k_2}(0)), \dots)$ ,  $p(1) = p$ , as well as

- if  $\zeta = 0$ ,  $p \in V^T$ , then  $\Phi = Q$  for any mapping  $Q$  of the form (15),
- if  $\zeta = 0$ ,  $p \notin V^T$ , then  $\Phi = Q$  for any mapping  $Q$  of the form (15) determined by vectors satisfying (17), and
- if  $\zeta \neq 0$ , then  $\Phi$  is the mapping of the form (21).

The existence of equilibrium in economy  $\mathcal{E}(1)$  is the consequence of Lemma 1 or 2. Based on the above, the structure  $\Gamma_0 = (M(0), \mu_0, h_0)$  is the economic mechanism in the sense of Hurwicz, resulting in equilibrium in economy  $\mathcal{E}(1)$ .

If, for at least one producer  $b$ ,  $Y^b(1) \not\subset \cup_{b \in B} Y^b(0)$ , then mechanism  $\Gamma_0$  by Theorem 1 is innovative. If, for every producer  $b$ ,  $Y^b(1) \subset \cup_{b \in B} Y^b(0)$ , then mechanism  $\Gamma_0$  is imitative (see Definition 4). If, at least for one producer  $b$ ,  $Y^b(0) \not\subset V$ , then mechanism  $\Gamma_0$  is a mechanism of destruction (Definition 5). Hence, if  $\Gamma_0$  is an innovative and destructive mechanism, then it is a mechanism of creative destruction (Definition 6).

In most cases, there are infinitely many solutions for systems of Equations (13), (14), and (17); hence, there are infinitely many mechanisms leading to equilibrium in economy  $\mathcal{E}(1)$  under the assumptions of Theorem 1. In summary, based on Theorem 1, it can be concluded that, in many cases, mechanisms of creative destruction (Definition 6) can lead to equilibrium. However, because the mechanisms defined in the proof of Theorem 1 are not all innovative, a destructive mechanism (Definition 5) can lead the economic system elaborated to equilibrium.

Note also that if  $p(0) = p$ , then within the mechanisms determined in the proof of Theorem 1, producers' profits and consumers' budget sets and preferences remain the same. If  $p(0) = \alpha p$ , for  $0 < \alpha < 1$ , and the profit of producer  $b$  at time  $t = 0$  is positive, then as a result of the mechanisms determined in the proof of Theorem 1, the profit of producer  $b$  at time  $t = 1$  is greater than that at time  $t = 0$ . Due to the above, the budget sets of some consumers are greater. Hence, in the above cases, the destructive mechanism defined in the proof of Theorem 1 can be considered in the category of qualitative mechanisms, i.e., the mechanisms in which, at time  $t = 1$ , the economic

positions of economic agents would not be worse off due to the given criterion than they were at time  $t = 0$  (compare with Lipieta and Malawski 2016).

Now we present the next theorem:

**Theorem 2.** If the assumptions of Lemma 3 are satisfied, then there exists a destructive mechanism  $\Gamma_0$ , which results in equilibrium in economy  $\mathcal{E}(1)$ , in which condition (22) is valid.

**Proof.** Components  $E(0)$ ,  $M(0)$ ,  $Z$ , and  $\mu_0$  are defined in the same way as in the proof of Theorem 1; However, in the definition of outcome function  $h_0$ , we have:  $p(1) = p$ ,  $x^k(1) = x^k(0)$  for  $k \in K$ ,  $y^k(1) = y^k(0)$  for  $k \in K \setminus \{b_{n_0+1}\}$ , and  $y^{b_{n_0+1}}(1) = \zeta$ . By applying Lemma 3, the thesis of the theorem is obtained.

If  $p(0) = p$  and the assumptions of Lemma 3 are satisfied, then economy  $\mathcal{E}(0)$  differs from economy  $\mathcal{E}(1)$  only in the activity of producer  $b_{n_0+1}$ . Producer  $b_{n_0+1}$  is inactive in economy  $\mathcal{E}(0)$  and active in economy  $\mathcal{E}(1)$ , but his economic position at time  $t = 1$  is worse than that at time  $t = 0$ . Hence, the mechanism  $\Gamma_0$  defined in the proof of Theorem 2 is the destructive mechanism (see Definition 5). However, if a new producer appearing on the market within the mechanism defined in the proof of Theorem 2 is an innovator, then that mechanism is a mechanism of creative destruction (Definition 6). Based on Theorem 2, such mechanism leads to equilibrium in economy  $\mathcal{E}(1)$ .

## 6. Conclusions

The main results of this research indicate the dual role of destructive mechanisms, which can lead to equilibrium in the economic system through either imitative or innovative processes. In the latter case, because a destructive mechanism is accompanied by innovative processes, then it can be interpreted as a mechanism of creative destruction in the sense of Schumpeter.

In contrast to the studies of Aghion and Howitt (1992, 1998), the present work showed that, in some cases, creative destruction can both generate innovativeness and lead to equilibrium through the elimination of worse or harmful commodities and processes. Some of the presented mechanisms also confirm the claims of Shionoya (2007, 2015) and Andersen (2009) about the existence of mechanisms adapting innovations and directing the economic system toward equilibrium.

The approach proposed in this study significantly differs from the traditional models often cited in the literature (see, for example, Aghion and Howitt 1992, 1998; Daron Acemoglu 2009; Romer 2012) on economic development. Axiomatization of the mechanisms of economic evolution in the Hurwicz apparatus showed the significant role of information and the way it is exchanged during innovative processes, as well as, in contrast to Schumpeter's ideas, a large complexity of mechanisms emerging in the process of evolution of the economy. The diversification of the modeled mechanisms also reveals the complexity of economic processes and their results (outcomes). In the set of outcomes of an innovative mechanism, the effects of creative destruction

are shown, besides new commodities, technologies and organizational structures, the old, unattractive products disappearing from the market.

The modeling of mechanisms of Schumpeterian evolution in this research showed the positive properties, from the producers' and consumers' points of view, of most of the examined mechanisms. In many cases, it provides the opportunity to identify the qualitative mechanisms from the set of possible mechanisms. The mechanisms that result in innovative changes differ not only in the economic environments, in the message spaces, and in the sets of outcomes but also in the sets of variables that characterize or will characterize the economic entities; this is the result of economic agents applying innovative changes to their routine activities. The identification of an optimal destructive mechanism remains within our research direction.

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