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Nonlinearity and Fractional Integration in the US Dollar/Euro Exchange Rate

Summary: This paper examines the nonlinear behavior and the fractional integration property of the US dollar/euro exchange rate over the period from January 1999 to August 2010 by extending the procedure of Peter M. Robinson (1994) to the case of nonlinearity. First, using the approach developed by Mehmet Caner and Bruce E. Hansen (2001), we investigate the possible presence of nonlinearity in the series through the estimation of a two-regime threshold autoregressive model. After finding nonlinearity, we also allow for disturbances to be fractionally integrated based on the different versions of Robinson (1994) tests. The findings show that the US dollar/euro exchange rate follows a stationary process with a weak evidence for long memory.

Key words: Nonlinear behavior, Long memory, Exchange rate.

JEL: C22.

The nonlinearity and nonstationarity properties of a time series has gained increased attention in the literature. In this context, extensive econometric research over the last years has focused on important issues, such as stock exchange and foreign currency, in economics and finance. The idea that real exchange rates are nonlinear has a long history. There are several economic reasons including transactions costs, central bank interventions, and the existence of limits to speculation, for considering nonlinearity in exchange rate data (Derek Bond, Michael J. Harrison, and Edward J. O'Brien 2009). Markov switching models (Charles Engel and James D. Hamilton 1990) and smooth transition autoregressive models (Lucio Sarno, Giorgio Valente, and Leon L. Hyginus 2004; Richard T. Baillie and Rehim Kilic 2005) are used in the literature to model nonlinearities. Because these approaches assume that the form of nonlinearity is known, Bond, Harrison, and O'Brien (2007) use three-step random field regression analysis of James D. Hamilton (2001) to explore the nature of nonlinearity.

On the other hand, another strand of the literature is the possibility of long memory or long-term dependence in the time series. It is well known that there is persistent temporal dependence even between distant observations if a series exhibits long memory. Presence of long memory dynamics causes nonlinear dependence in the first moment of the distribution and, hence, potentially predictable component in the dynamics of the series (John T. Barkoulas and Christopher F. Baum 1997). Fractionally integrated processes can give rise to long memory (Benoit B. Mandelbrot 1977; Clive W. J. Granger and Roselyne Joyeux 1980; Jonathan R. M. Hosking 1981).

In recent years, there have been substantial developments in modeling long memory processes and also in the mainly unrelated area of modeling nonlinearity. However, there has been relatively little consideration of the issue of combining and distinguishing between these types of processes (Ballie and George Kapetanios 2005). Francis X. Diebold and Atsushi Inoue (2001) and Kapetanios and Yongcheol Shin (2003) consider the possibility of confusing nonlinearity and long memory. Diebold and Inoue (2001) show how a process with Markov switching regime changes can be mistaken for a long memory process. Kapetanios and Shin (2003) suggest a formal test for distinguishing between nonstationary long memory and nonlinear geometrically ergodic processes in small samples. Dick Van Dijk, Timo Terasvirta, and Philips H. Franses (2002) address the possibility that a process may exhibit both long memory dynamics and nonlinearity in the short memory dynamics, and they consider long memory and exponential smooth transition autoregressive model to represent the US unemployment rate. As a different approach, Guglielmo M. Caporale and Luis A. Gil-Alana (2007) model unemployment as a nonlinear process and allow for disturbances to be fractionally integrated. They introduce both fractional integration and nonlinearities simultaneously into the same framework based on Robinson (1994) tests.

Following Caporale and Gil-Alana (2007), we examine nonlinear behavior of the US dollar/euro exchange rate series instead of assuming linearity and also allow for the possibility of fractional values instead of using integer integration orders. The difference of this paper is that we test nonlinearity using the Wald test suggested by Caner and Hansen (2001) and construct a two-regime threshold autoregressive (TAR) model for the US dollar/euro exchange rate that allows us to derive endogenous threshold effects. Then, the Robinson (1994) tests are applied to the residuals obtained from the corresponding TAR model for fractional integration. This paper contributes to the literature using two-regime TAR model approach of Caner and Hansen (2001) and Robinson (1994) tests in the same analysis for nonlinearity and fractional integration properties of the US dollar/euro exchange rate, respectively.

The structure of the paper is as follows: Section 1 presents the econometric methodology used in the paper, Section 2 discusses the data and empirical results, and Section 3 concludes.

1. Robinson Tests Under the Case of Nonlinearity

To examine the time series behavior, we take into account the possible nonlinearity and fractional integration properties of the US dollar/euro exchange rate in the same analysis following Caporale and Gil-Alana (2007). For this purpose, the nonlinearity is investigated using a TAR model that allows to derive endogenous threshold effects, and the fractional integration property is determined using different versions of the Robinson (1994) tests. In this section, we consider that it is better to describe Robinson (1994) tests under the case of linearity before the case of nonlinearity.

Robinson (1994) considers the following regression model:

$$y_t = \beta' z_t + x_t, \quad t = 1, 2, ... \tag{1}$$

where y_t is the observed time series for $t = 1, 2, ...T$, $\beta = (\beta_1, ..., \beta_k)'$ is a $(k \times 1)$ vector of unknown parameters, and z_t is a $(k \times 1)$ vector of deterministic regressors such as an intercept or a linear trend. The regression errors x_t can be explained as follows:

$$(1-L)^d x_t = u_t, \quad t = 1, 2, ... \tag{2}$$

where L is the lag operator and u_t is an $I(0)$ process. Here, d can take any real value. If $d = 0$ in Eq. (2), $x_t = u_t$, and a "weakly autocorrelated" x_t is allowed for. When $d > 0$, x_t is said to be "strongly autocorrelated" or "strongly dependent". Clearly, the unit root case corresponds to $d = 1$ in (2). If $d > 0$, x_t is said to be long memory (Granger and Joyeux 1980; Hosking 1981). If $0.5 < d < 1$, the process is nonstationary and exhibits long memory, whereas the process is stationary and exhibits long memory, if $0 < d < 0.5$. It is important to note that when $d < 0.5$, the process is stationary and mean reverting with the effect of the shocks dying away in the long run and when $0.5 \le d$, the process is nonstationary even if the fractional parameter is significantly less than 1.

Robinson (1994) proposes Lagrange multiplier (LM) test to test unit roots and other forms of nonstationary hypotheses, embedded in fractional alternatives. The null hypothesis of the test can be seen below:

$$H_0 : d = d_0 \tag{3}$$

Specifically, the test statistic is given by the following:

$$\hat{r} = \frac{T^{1/2}}{\hat{\sigma}^2} \hat{A}^{1/2} \hat{a} \tag{4}$$

where T is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j) \ ; \ \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j) \ ;$$

$$\hat{A} = \frac{2}{T} \left(\sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right)$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right| ; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}_j) \ ; \quad \lambda_j = \frac{2\pi j}{T} ; \quad \hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau)$$

where $I(\lambda_j)$ is the periodogram of u_t, and T^* is a compact subset or the Euclidean space.

The main advantage of the Robinson (1994) procedure is that it tests unit and fractional roots with a standard null limit distribution, which is unaffected by inclusion or not of deterministic trends. Under certain regularity conditions, Robinson (1994) shows that the test statistic is

$$\hat{r} \rightarrow_d N(0,1) \text{ as } T \rightarrow \infty. \tag{5}$$

Thus, a one-sided $100\alpha\%$ level test of Eq. (3) against the alternative $H_1: d > d_0$ is given by the rule "Reject H_0 if $\hat{r} > z_\alpha$" where the probability that a standard normal variate exceeds z_α is α, and conversely, a one-sided $100\alpha\%$ level test of Eq. (3) against the alternative $H_1: d < d_0$ is given by the rule "Reject H_0 if $\hat{r} < -z_\alpha$". Empirical applications of the test with this version and other versions can be found in Gil-Alana and Robinson (1997, 2001) and Gil-Alana (1999, 2000, 2001, 2002).

In our analysis, we follow the paper of Caporale and Gil-Alana (2007), which extends the Robinson (1994) procedure to the case of nonlinear regression models. As a difference, we use a two-regime TAR model suggested by Caner and Hansen (2001) that allows to derive endogenous threshold effects in the series, instead of Eq. (1). Recent work by Caner and Hansen (2001) presents some new results on the TAR model introduced by Howell Tong (1978). They develop new tests for threshold effects and estimate the threshold parameter. To investigate whether there exists a nonlinear behavior in the data, the corresponding model can be specified as follows:

$$\Delta \mathrm{ER}_t = \theta_1' x_{t-1} 1_{\{Z_{t-1} \langle \lambda\}} + \theta_2' x_{t-1} 1_{\{Z_{t-1} \geq \lambda\}} + \varepsilon_t \tag{6}$$

where ER is the US dollar/euro exchange rate series for t = 1,...,T, $x_{t-1} = (ER_{t-1} r_t' \Delta ER_{t-1} \ldots \Delta ER_{t-k})'$, $1_{\{.\}}$ is the indicator function, ε_t is an independently and identically distributed (*IID*) error term, $Z_t = \mathrm{ER}_t - \mathrm{ER}_{t-m}$ for some delay parameter, m $\geq$ 1 is the threshold variable, r_t is a vector of deterministic components including intercept and a possible linear time trend, and k $\geq$ 1 is the autoregressive order. The threshold parameter (λ) is unknown and takes values in the interval $\lambda \in \Lambda = [\lambda_1, \lambda_2]$, where λ_1 and λ_2 are picked so that $P(Z_t \leq \lambda_1) = \pi_1 > 0$ and $P(Z_t \leq \lambda_2) = \pi_2 < 1$. The components of θ_1 and θ_2 are as follows:

$$\theta_1 = \begin{pmatrix} \rho_1 \\ \beta_1 \\ \alpha_1 \end{pmatrix}, \qquad \theta_2 = \begin{pmatrix} \rho_2 \\ \beta_2 \\ \alpha_2 \end{pmatrix},$$

where $\rho_1, \rho_2, \beta_1, \beta_2$ are scalars and α_1 and α_2 are $k \times 1$ vectors. Thus, (ρ_1 ,ρ_2) are slope coefficients on ER_{t-1}, (β_1 ,β_2) are the slope coefficients on the deterministic components r_t, and (α_1,α_2) are the slope coefficients on $(\Delta \mathrm{ER}_{t-1},...,\Delta \mathrm{ER}_{t-k})$ in the two regimes. The TAR model is estimated using least squares (LS). By estimating the TAR model in Eq. (6), we can investigate whether there exists threshold effect in the data. Standard Wald statistic, $W_T = W_T(\hat{\lambda}) = \sup_{\lambda\in\Lambda} W_T(\lambda)$, proposed by Caner and Hansen (2001), is used to test the null hypothesis of no threshold effect that $H_0 : \theta_1 = \theta_2$, against the alternative of a threshold effect. If the null hypothesis cannot be rejected, then there is no threshold, in which case the vectors of coefficients θ are identical between two regimes ($\theta_1 = \theta_2$). Caner and Hansen (2001) indicate that $\sup_{\lambda\in\Lambda} W_T(\lambda)$ has a nonstandard asymptotic null distribution and suggest a bootstrap method to compute asymptotic critical values and p-values. After finding a threshold effect in the data, we apply Robinson (1994) tests to the residuals obtained from the TAR model in the second step of the analysis.

2. Data and Empirical Results

This paper uses the data $ER_t = \log(S_t / S_{t-1})$ where S_t is the weekly US dollar/euro exchange rate from January 1999 to August 2010 yielding T = 605 observations. It is important to note that $ER_t = \log(S_t / S_{t-1})$ is the return series. The data are obtained from the Central Bank of the Republic of Turkey.

2.1 Nonlinearity Test Results and TAR Model Estimations

In the first step of the analysis, the null hypothesis of linearity against the alternative of threshold effects are tested for ER series. For this purpose, the Wald test W_T statistics are calculated as reported in the previous section using Gauss program codes provided on Bruce Hansen's Web page (www.ssc.wisc.edu/~bhansen/progs/progs_threshold.html). The results of the Wald test, the bootstrap critical values generated under the conventional significance levels, and the bootstrap p-values for threshold variables such that $Z_t = \mathrm{ER}_t - \mathrm{ER}_{t-m}$ for delay parameters m from 1 to 4 can be seen in Table 1.

Table 1 The Results of Wald Test for Threshold Effects

m	W_T	Bootstrap critical values			Bootstrap p-values
		10%	5%	1%	
1	**62.8**	30.9	34.0	40.8	0.000
2	47.9	30.5	33.5	40.4	0.000
3	53.9	30.5	33.8	39.6	0.000
4	44.7	30.8	33.8	40.3	0.003

Note: *p-values are calculated from 10,000 replications.

Source: Author's estimations.

It is seen from the table that all the statistics are significant at the 1% significance level and support the existence of threshold effect or nonlinearity. Caner and Hansen (2001) propose selecting m that minimizes the residual variance. This is equivalent to selecting m that maximizes the value of the W_T. As can be seen from the table, the W_T statistic is maximized (W_T = 62.8) when m = 1. When the bootstrap p-values are recalculated for m = 1, we find it as 0.000 again. Following this result, which gives a strong support to the existence of a threshold effect in the US dollar/euro exchange rate, we estimate two-regime TAR model for m = 1 using least squares estimation method. The estimation results are given in Table 2.

Table 2 The Estimation Results of TAR Model

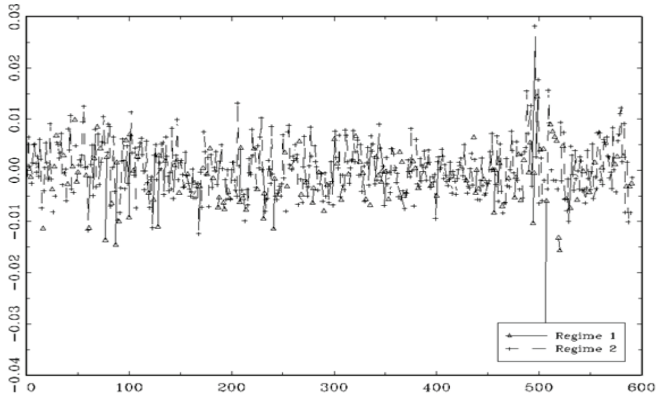
	Threshold estimate: $\hat{\lambda}$ = -0.0044			
	$Z_{t-1} < \hat{\lambda}$		$Z_{t-1} \geq \hat{\lambda}$	
Regressors	**Estimate**	**Std. error**	**Estimate**	**Std. error**
Constant	-0.003***	0.001	0.000	0.000
ER_{t-1}	-1.050***	0.268	-0.616***	0.131
ΔER_{t-1}	-0.128	0.238	-0.163	0.136
ΔER_{t-2}	0.072	0.252	-0.178	0.123
ΔER_{t-3}	0.203	0.237	-0.139	0.116
ΔER_{t-4}	0.290	0.207	-0.188*	0.114
ΔER_{t-5}	0.076	0.197	-0.111	0.105
ΔER_{t-6}	0.357**	0.172	-0.177*	0.100
ΔER_{t-7}	0.048	0.166	-0.149*	0.093
ΔER_{t-8}	-0.049	0.157	-0.121	0.087
ΔER_{t-9}	-0.155	0.147	-0.111	0.077
ΔER_{t-10}	-0.096	0.123	-0.059	0.070
ΔER_{t-11}	-0.285**	0.105	0.084	0.059
ΔER_{t-12}	-0.188**	0.076	0.022	0.049

Note: ***,** and * indicate statistically significance at the 1%, 5% and 10% significance levels, respectively.

Source: Author's estimations.

The point estimate of threshold ($\hat{\lambda}$) is found as -0.0044. This result indicates that TAR model splits the regression into two regimes, depending on whether the variable $Z_{t-1} = ER_{t-1} - \mathrm{ER}_{t-2}$ lies above or below -0.0044. The illustration of the two regimes can be seen in Figure 1.

The first regime is the case of $Z_{t-1} <$ -0.0044, which occurs when the deviation of the ER falls, remains constant, or rises by less than -0.0044. The second regime is the case of $Z_{t-1} \geq$ -0.0044, which occurs when the deviation of the ER rises by more than -0.0044. The results show that around 23% observations fall into the first regime, whereas around 77% of the observations fall into the second regime. After finding threshold effects in the US dollar/Euro exchange rate and constructing TAR model, we apply Robinson (1994) tests to determine the fractional integration properties of the data in the second part of the analysis.



Source: The author.

Figure 1 The ER Data, Classified by Threshold Regime

2.2 Robinson (1994) Test Results

Following the paper of Caporale and Gil-Alana (2007), which extends the Robinson (1994) procedure to the case of nonlinear regression models, we apply the Robinson (1994) test procedure in the case of nonlinearity and estimate a two-regime TAR model instead of Eq. (1) as reported before. Then, we calculate the one-sided test statistics $\hat{r}$ for the residuals obtained from the TAR model with $d_0 = 0, 0.05, 0.10, 0.15, 0.20, 0.25, ..., 0.50, ..., 1$. Under the null hypothesis $H_0 : d = d_0$, the $I(0)$ disturbances are modeled in both white noise and AR(1) processes for the case with a constant and the case with a linear trend. The results are reported in Table 3.

Table 3 Robinson Test Results for the Residuals from TAR Model

	White noise disturbances		AR(1) disturbances	
d_0	An intercept	A linear trend	An intercept	A linear trend
0	**-0.142****	**-0.137****	**-0.368****	**-0.360****
0.05	-1.521**	-1.517**	-3.449	-3.444
0.10	-2.693	-2.691	-6.081	-6.077
0.15	-3.693	-3.692	-8.329	-8.327
0.20	-4.550	-4.549	-10.257	-10.255
0.25	-5.287	-5.287	-11.920	-11.919
0.30	-5.925	-5.925	-13.366	-13.365
0.35	-6.481	-6.480	-14.636	-14.635
0.40	-6.967	-6.966	-15.761	-15.761
0.45	-7.394	-7.394	-16.770	-16.770
0.50	-7.773	-7.773	-17.683	-17.683
0.55	-8.110	-8.110	-18.519	-18.519
0.60	-8.411	-8.411	-19.290	-19.290
0.65	-8.681	-8.681	-20.007	-20.007
0.70	-8.926	-8.926	-20.681	-20.680
0.75	-9.147	-9.147	-21.316	-21.316
0.80	-9.349	-9.349	-21.920	-21.920

0.85	-9.533	-9.533	-22.497	-22.497
0.90	-9.702	-9.702	-23.051	-23.051
0.95	-9.858	-9.858	-23.584	-23.584
1.00	-10.002	-10.002	-24.100	-24.100

Note: The smallest value across the different values of d_0. ** indicates nonrejection values of the null hypothesis at the 95% significance level.

Source: Author's estimations.

In the table, for a given d_0, significantly negative values of $\hat{r}$ are consistent with the orders of integration smaller than d_0. A notable feature is the fact that $\hat{r}$ monotonically decreases with d_0. This is something to be expected because it is a one-sided test statistic. The results show that H_0 hypothesis cannot be rejected for d_0 = 0 and 0.05 in both cases with a constant and a linear trend if the disturbances are white noise. On the other hand, if the disturbances are assumed to be an AR(1) process, H_0 cannot be rejected for d_0 = 0 in cases with a constant and with a linear trend. It is clear that the lowest statistic across the different values of d_0 occurs when $d_0 = 0$ in both disturbances. These findings show that the US dollar/euro exchange rate follows a stationary process, and there is a weak evidence of long memory only for white noise disturbances.

3. Conclusions

Following the paper of Caporale and Gil-Alana (2007), which examines both fractional integration and nonlinearities simultaneously in the same framework based on Robinson (1994) tests, this paper investigates the nonlinear behavior of the US dollar/euro exchange rate from January 1999 to August 2010 instead of assuming linearity and also allows for the possibility of fractional values instead of using integer orders of integration. As a difference, we use a two-regime TAR model approach of Caner and Hansen (2001) for nonlinearity and different versions of Robinson (1994) tests for fractional integration. In the first step, the nonlinearity of the series is tested using Wald test as suggested by Caner and Hansen (2001), and a two-regime TAR model that allows to derive endogenous threshold effects is estimated. The results of the TAR model show that the point estimate of threshold is -0.0044, and around 23% of observation fall into the first regime, which is the case of $Z_{t-1} <$ -0.0044, whereas around 77% of observations fall into the second regime, which is the case of $Z_{t-1} \geq$ -0.0044. In the second step of the analysis, we apply different versions of Robinson tests to the residuals obtained from the TAR model. Under the null hypothesis $H_0 : d = d_0$, the disturbances are modeled in both white noise and AR(1) processes, and one-sided test statistics are calculated. The findings indicate that the US dollar/euro exchange rate follows a stationary behavior in both processes, and there is only evidence for long memory in white noise process.

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