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Preliminary report

Yu-Chuan Chen
Chihlee University of Technology,
Department of Finance,
Taiwan
✉ ycchen@mail.chihlee.edu.tw

Pin Ruei Lee
Hontai Life Insurance Co., Ltd.,
Marketing and Planning Department,
Taiwan,
R.O.C.
✉ leepin38@gmail.com

Yung-Ho Chiu
Corresponding author
Soochow University,
Department of Economics,
Taiwan
✉ echiu@scu.edu.tw

The Efficiency of Non-Homogeneity Security Firms in Taiwan

Summary: Data envelopment analysis (DEA) is a nonparametric technique for determining the efficiency of a homogeneous set of decision-making units (DMUs). There are two common problems with traditional DEA. First, traditional DEA fails to adequately distinguish the efficiency DMUs. Second, the DMUs within the same industry are non-homogeneous. This study aims to develop a system-ranking-efficiency model to solve the problems of non-homogeneity and efficiency ranking for DMUs in the same group. The proposed system-ranking-efficiency model is based on the concept of boundary change and considers the efficiency DMUs with the greatest influence on the boundary as the most important and, thus, as the ones that should have the highest efficiency ranking. The model is applied in the Taiwan securities industry, in which it was found to successfully rank all the DMUs.

Key words: Data envelopment analysis, Non-homogeneity, System-DEA, Ranking.

JEL: D24, G23.

Based on the literature, Data envelopment analysis (DEA) provides a basis for the efficiency ranking of decision-making units (DMUs). There are two common problems with DEA. First, when the efficiency value for several DMUs is 1, DEA fails to adequately distinguish among these efficiency DMUs. Second, the DMUs within an industry can be in different systems frontiers; thus, when DEA is adopted, the performance assessment cannot be carried out for the whole industry. Subsequently, some approaches for differentiation need to be applied. The present study describes a system-ranking-efficiency model that aims to address the problems of non-homogeneity and efficiency ranking for the DMUs in an industry.

There are three primary activities that securities firms can engage in: securities brokerage, securities dealership, and securities underwriting. Firms that engage in the three aforementioned activities simultaneously are known as integrated securities firms, whereas those that engage in only one or two areas are referred to as professional securities firms. Integrated securities firms must have at least one billion Yuan in capital. Hence, in terms of company size and business scale, integrated securities firms are usually much larger than professional securities firms. There are definite and palpable differences between these two kinds of firms; their market advantages and the niches they occupy within the industry differ, and thus differences exist in the nature of these firms in the industry. Therefore, in assessing the performance of integrated securities firms within the industry in Taiwan, the problem of the heterogeneity of firms must be considered.

Due to the differences between the two types of securities firms, it seems inappropriate to evaluate their performances at the same time. To resolve such a pragmatic issue, a system-ranking-efficiency DEA model is proposed to address the problems of non-homogeneity and efficiency ranking for the DMUs in the Taiwan securities industry. This approach makes it possible to analyze the efficiency ranking in the Taiwan securities industry correctly and effectively. The present report is organized as follows: Section 1 provides a literature review, Section 2 presents the research methodology, Section 3 gives an analysis of the empirical results, and Section 4 provides the conclusions.

1. Literature Review

DEA was first proposed in 1978 by Abraham Charnes, William W. Cooper, and Edwardo L. Rhodes (1978), whose research assessed efficiency by using a method for studying constant returns to scale called the CCR model. Later, Rajiv D. Banker, Charnes, and Cooper (1984) developed a model for variable returns to scale (VRS), called the BCC model. Regardless of whether the CCR or the BCC model is used, however, when multiple efficiency DMUs exist, the model fails to differentiate among these DMUs, thus causing problems in their ranking. Moreover, the production points within the systems frontier differ in efficiency values due to their different locations.

For the ranking of multiple efficiency DMUs, it is necessary that their entire original efficiency values are equal to 1. Therefore, researchers have proposed a more conscientious and careful definition and criteria. Charnes, Cooper, and Robert M. Thrall (1986), and Lawrence M. Seifordand and Thrall (1990) divided efficiency DMUs into three subsets. DMUs with strong efficiency are at the extreme points on the efficiency surface. Those with weak efficiency lie at the boundary points on the extended portion of the efficiency surface and can be expressed as a linear combination of strong efficiency DMUs. Alternatively, DMUs that are neither at the extreme points on the efficiency surface nor at the boundary points on the extended portion of the efficiency surface lie at the remaining boundary points on the efficiency surface.

To resolve the efficiency ranking problem, the modified DEA (MDEA) proposed by Per Andersen and Niels Christian Petersen (1993) exclude DMU from the reference set and then carry out estimations of the efficiency values. The super-efficiency value for DMU in a strong efficiency set is larger than 1, whereas the efficiency value for DMU in a weak efficiency set is 1. Therefore, the MDEA model can distinguish between a weak efficiency set and a strong efficiency set of DMUs. However, Thrall (1996) found that super-efficiency DEA under VRS could be infeasible. Consequently, the super-efficiency value could be both infinitely large and unpredictable. When the efficiency DMU fails to predict the problem, it would be unable to correctly rank the DMUs. Thus, resolving this infeasibility is a subject that has been much emphasized by researchers, such as Joe Zhu (1996), Yao Chen (2003), C. A. Knox Lovell and A. P. B. Rouse (2003), Peter Bogetoft and Jens Leth Hougaard (2004), Gholam Reza Jahanshahloo et al. (2007). These authors all proposed methods to resolve the issue of obtaining an efficiency value equal to 1. The feasibility, as well as the efficiency value, of each method is worthy of discussion.

DEA is a nonparametric technique for determining the efficiency of a homogeneous set of DMUs. Another problem with the DEA model is that the DMUs come from a single group with different frontiers. When there are two or more production frontiers existing in an industry, DEA is likely to overestimate the programmatic efficiency. One strategy to solve this problem is to separate DMUs into homogeneous groups if they belong to different systems frontiers. However, the assessment with two categories also fails to cover the whole aspect of the industry, and large numbers of DMUs are needed to apply this approach. Therefore, Kaoru Tone (1993), Patrick L. Brockett and Boaz Golany (1996), David A. Haas and Frederic H. Murphy (2003), Gary Simpson (2005), and Rolf Färe and Shawna P. Grosskopf (2006) applied some measures to adjust for non-homogeneity if the groups have different frontiers.

In practical application, the securities industry typically has the characteristic of non-homogeneity. DEA is widely used for performance assessment of financial industries, such as by Claudia Girardone, Philip Molyneux, and Edward P. M. Gardener (2004), Ta-Cheng Chang and Yung-Ho Chiu (2006), Chiu et al. (2008), Chiu and Yu-Chuan Chen (2009), and Chiu, Chen, and Xue-Jie Bai (2011). Nevertheless, Nicholas Apergis and Effrosyni Alevizopoulou (2011), Saadet Kasman and Adnan Kasman (2011) applied the DEA method to estimate banking efficiency. Yin-Ching Jan and Mao-Wei Hung (2003), Randy I. Anderson et al. (2004), and Ruiyue Lin and Zhiping Chen (2008) also applied DEA to assess the relative performance of mutual funds. However, the literature on the application of DEA in the securities industry is limited. Hirofumi Fukuyama and William L. Weber (1999) used DEA to evaluate the efficiency of Japanese securities firms during the period of 1988 to 1993. Wei David Zhang, Shuo Zhang, and Xueming Luo (2006) applied DEA to assess the performance of the major US securities firms. Also, Chin-Yi Fang and Jin-Li Hu (2009, 2010) used zero-sum gains DEA to estimate Taiwan securities firms. With the exception of Fukuyama and Weber (1999), Zhang, Zhang, and Luo (2006), and Fang and Hu (2009, 2010), few researchers have addressed the non-homogeneity problem in securities firms, the resolution of which is essential.

The ranking of DMUs with an efficiency value of 1 has always been an issue with DEA. The non-homogeneity problem of the companies within an industry has also gradually gained attention. In previous literature, these two problems were discussed separately; they have never been considered together. Consequently, an efficiency model for non-homogeneous systems also needs to consider the ranking problem when all efficiency values are 1. The present study focuses on the Taiwan securities industry, considers the non-homogeneity problem existing in the industrial samples, and studies the super-efficiency model in non-homogeneous systems.

2. Research Methodology

Based on past research experience, attention should be given to the suitability and limitation of the efficiency model used in performance assessment.

2.1 Based Model: BCC Model

Banker, Charnes, and Cooper (1984) developed a model for variable returns to scale called the BCC model. Assuming that x_{ih} denotes the i-th ($i = 1, ..., I$) inputs of h-th ($h = 1, ..., H$) DMU, and Y_{mh} refers to the m-th ($m = 1, ..., M$) outputs of the h-th DMU, when estimating the performance of DMU_h, the model would select the most advantageous weight λ_h ($h = 1, ..., H$) to achieve the maximum efficiency score. The efficiency for the input from any DMU_k can be estimated by the following BCC model:

【BCC Model】 —

$$\min_{\theta_k, \lambda_1, \lambda_2, \ldots, \lambda_H} \quad E_k = \theta_k$$

$$s.\ t.\ \theta_k x_{ik} \geq \sum_{h=1}^{H} \lambda_h x_{ih}\ , \quad i = 1, 2, \ldots, I;\ h = 1, 2, \ldots, H;$$

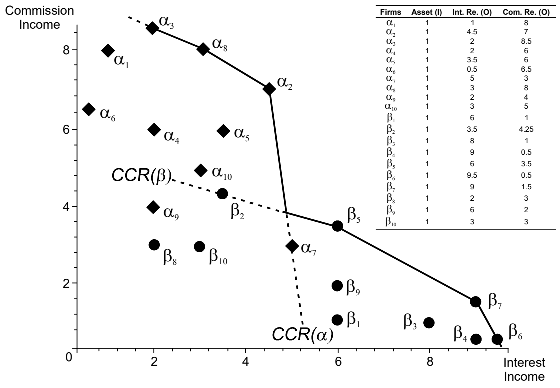
$$y_{mk} \leq \sum_{h=1}^{H} \lambda_h y_{mh}\ , \quad m = 1, 2, \ldots, M;\ h = 1, 2, \ldots, H; \tag{1}$$

$$\sum_{h=1}^{H} \lambda_h = 1$$

$$\lambda_h \geq 0, h = 1, 2, \ldots, H.$$

2.2 Non-Homogeneity Problem: System-BCC Model

The system-BCC model is used to simulate an example with two outputs and one input to describe the situation of non-homogeneity for the DMUs in an industry. Assuming that there are 20 DMUs from the same industry, their outputs are interest income and commission income, whereas their input is total assets. The longitudinal and lateral axes denote the actual outputs, as shown in Figure 1.



Firms	Asset (I)	Int. Re. (O)	Com. Re. (O)
α_1	1	1	8
α_2	1	4.5	7
α_3	1	2	8.5
α_4	1	2	6
α_5	1	3.5	6
α_6	1	0.5	6.5
α_7	1	5	3
α_8	1	3	8
α_9	1	2	4
α_{10}	1	3	5
β_1	1	6	1
β_2	1	3.5	4.25
β_3	1	8	1
β_4	1	9	0.5
β_5	1	6	3.5
β_6	1	9.5	0.5
β_7	1	9	1.5
β_8	1	2	3
β_9	1	6	2
β_{10}	1	3	3

Source: Tone (1993).

Figure 1 Boundary for a Non-Homogeneous System

In Figure 1, α system represents "companies in the service industry", whereas β system represents "companies in the investment industry". A system frontier can be found for the above system α and β, respectively, and it is possible to observe the difference boundaries between the two DMUs. Without specifying the difference between the two systems, the systems frontier for all the companies in the whole industry consists of these six points: $\alpha_3, \alpha_8, \alpha_2, \beta_5, \beta_7$, and β_6. By comparing the systems frontier for all DMUs and the individual systems frontiers for systems α and β, the following results can be obtained: for β_2, the best efficiency value for the business performance in the β system is 1; however, when the boundary is based on all the DMUs in the industry, β_2 does not fall within the systems frontier. Similarly, although α_7 in the α system already falls within the systems frontier, when the systems frontier is based on the whole industry, α_7 does not achieve efficiency.

Tone (1993) proposed the system-BCC model, which assumes that in the α system, the input matrix for the DMUs (denoted by DMU_α) is X_α and that in the β system, the input matrix for the DMUs (DMU_β) is X_β; then, the efficiency for the input from any DMU_k can be estimated by the following system-BCC model:

【System-BCC Model】 —

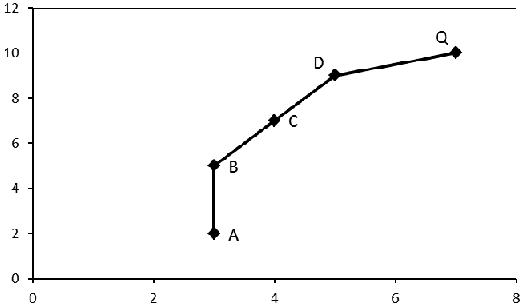
$$\begin{aligned}
&\underset{\theta_k,\lambda_1,\lambda_2,\dots,\lambda_H}{Min} \quad E_k = \theta_k \\
&s.\ t.\ \ \theta_k x_{ik} \ge \sum\nolimits_{h\in\alpha} \lambda_h x_{ih} + \sum\nolimits_{h\in\beta} \lambda_h x_{ih}\ , i = 1, 2, \dots, I; h = 1, 2, \dots, H; \\
&y_{mk} \le \sum\nolimits_{h\in\alpha} \lambda_h y_{mh} + \sum\nolimits_{h\in\beta} \lambda_h y_{mh},\ , \quad m = 1, 2, \dots, M; h = 1, 2, \dots, H; \\
&\sum\nolimits_{h\in\alpha} \lambda_h = Z_\alpha \\
&\sum\nolimits_{h\in\beta} \lambda_h = Z_\beta \\
&Z_\alpha, Z_\beta = 0 \text{ or } 1,\ Z_\alpha + Z_\beta = 1; \\
&\lambda_1, \dots, \lambda_H \ge 0; \theta_k \text{ without limitation on positivity and negativity.}
\end{aligned} \tag{2}$$

In Equation (1), Z_α and Z_β are grouping variables, defined as 1 or 0. They represent two different DMUs in an industry. After the simulation of the two systems frontiers of an industry, the systems frontier for the whole industry appears as the outermost contour for the two systems frontiers. Through (1) model setting, we can discuss the efficiency ranking for DMU_α in the α system and in the whole industry $(\alpha \cup \beta)$, as well as further understand the performance of DMU_α in the β system. For the same reason, the performance ranking of DMU_β in the α system can be obtained. The efficiency value for DMU_k in the system model θ_k^* (i.e., the smaller of θ_α and θ_β) is:

$$\theta_k^* = \min\{\theta_\alpha, \theta_\beta\}. \tag{3}$$

2.3 Efficiency Ranking Problem: Ranking-System Model

The theoretical basis for DEA is the use of efficiency DMUs as the boundary and the setting of the efficiency value to 1. Subsequently, other DMUs use the boundary as the target. The distance of each DMU from the boundary is its efficiency value. Therefore, the boundary is essential for the efficiency estimation. As shown in Figure 2, if the original DEA is used with the BCC or CCR model to estimate the DMUs efficiency values, all the five DMUs achieve an efficiency value equal to 1. Thus, it is impossible to obtain efficiency rankings. However, if we use the classification rules described by Charnes, Cooper, and Thrall (1986) and by Seiford and Thrall (1990), the efficiency rankings for the five DMUs would be: B, D, $Q \in E \Rightarrow C \in E'$ $\Rightarrow A \in F$; i.e., B, D, and Q have strong efficiency (E), C has semi-strong efficiency (E'), and A has weak efficiency (F).



Source: Created by the authors and referenced by Mei Xue and Patrick T. Harker (2002).

Figure 2 Classification of Efficiency DMUs

To further differentiate the efficiency rankings of the DMUs, various solutions have been proposed by several researchers, including Andersen and Petersen (1993), Zhu (1996), Chen (2003), Lovell and Rouse (2003), Bogetoft and Hougaard (2004), and Jahanshahloo et al. (2007). Jahanshahloo et al. (2007), who used the boundary mode as theoretical basis, proposed the exclusion of the efficiency DMUs and the use of the boundary change to determine the importance of the efficiency DMUs. The authors further estimated the efficiency values to prevent problems associated with infeasibility.

Jahanshahloo et al. (2007) proposed a ranking-system model in which a DMU with strong efficiency in the CCR or BBC model is denoted as SE. The original efficient frontier changes if DMU_b is SE, and this new efficient frontier without DMU_b moves closer to the inefficient DMUs. The SEDMU, which, when excluded from the reference set of all the other DMUs allows the efficient frontier to be closest to the inefficient DMUs, should be the most efficient SEDMU. Similarly to the classical

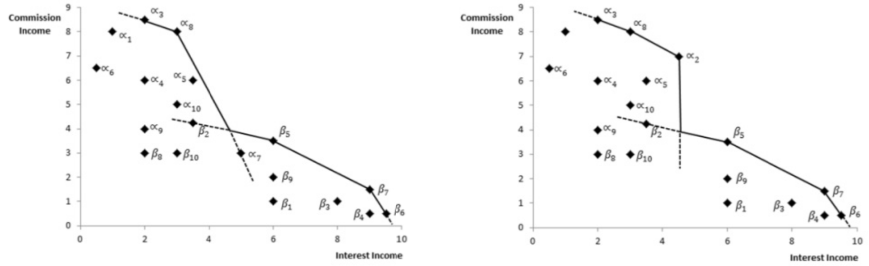
DEA models, the DMUs that belong to this “envelope” are the best performers; the DMU that influences the efficient frontier to move farther from the remaining data should be classified as the best. However, to apply the ranking-system model, the non-SE DMUs should be reevaluated.

【Ranking-System Model】 —

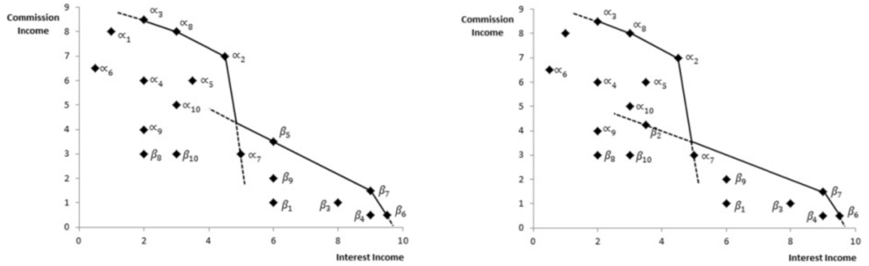
$$\begin{aligned}
&\min_{\theta,\lambda_1,\lambda_2,\ldots,\lambda_H} \partial_{a,b} = \theta \\
&s.\ t.\ \theta x_{ia} \geq \sum_{h\in H-\{b\}} \lambda_h x_{ih}\ ,\ i = 1, 2, \ldots, I; \\
&y_{ma} \leq \sum_{h\in H-\{b\}} \lambda_h y_{mh},\ ,\quad m = 1, 2, \ldots, M; \\
&\lambda_1,\ldots,\lambda_H \geq 0;\ \theta \text{ without limitation on positivity and negativity} \\
&a \in J_n\ ;\ b \in J_e\ ;\ h \in H-\{b\} \\
&J_n \text{ is the set of non-SEDMUs;} \\
&J_e \text{ is the set of SEDMUs.}
\end{aligned} \tag{4}$$

2.4 Modifying Non-Homogeneity and Super-Efficiency: System-Ranking-Efficiency Model

To improve the traditional DEA model that fails to consider the DMUs, address the non-homogeneity problem, and differentiate super-efficiency values, this study proposes a system-ranking-efficiency model based on the above system-BCC and ranking-system models. The proposed model reassesses the non-SE DMUs after differentiating the two non-homogeneous systems. The data shown in Figure 1 are used to explain the system-ranking-efficiency model in Figure 3, in which the companies in the industry are denoted as α and β. The companies on the boundary are the efficiency DMUs in each system. The efficiency DMU α_2 in the α system is removed, as shown in Figure 3(A). Then, Figure 3(A) is compared with the original boundary in Figure 1 to determine the boundary change. After deleting the efficiency DMU α_7 in the α system, as shown in Figure 3(B), different boundary changes can be obtained. Furthermore, the effect of deleting α_2 or α_7 on the boundary is clearly shown to be different. Similarity is found when comparing the efficiency DMUs in the β system after deleting β_2 and β_5, as shown in Figure 3(C) and (D), respectively. The system-ranking-efficiency model considers the DMUs with the greatest changes in boundary as the most important.



(A) deleting α2 (B) deleting α7



(C) deleting β2 (D) deleting β5

Source: Created by the authors and referenced by Tone (1993).

Figure 3 Explanation of the Graph in the System-Ranking-Efficiency Model

The bilateral model for the input-oriented system-ranking-efficiency model is as follows:

$$\text{【System-ranking-efficiency model】} -$$

$$\min_{\theta_k,\lambda_1,\lambda_2,\ldots,\lambda_H} \quad E_{a,b} = \theta_k$$

$$s.\ t.\ \ \theta_k X_{ik} \geq \sum_{\substack{h\in\alpha \\ h\in H_\alpha-\{b\}}} \lambda_h x_{ih} + \sum_{\substack{h\in\beta \\ h\in H_\beta-\{b\}}} \lambda_h x_{ih}\ ,\ \ i = 1, 2, \ldots, I;\ \ h = 1, 2, \ldots, H;$$

$$y_{mk} \leq \sum_{\substack{h\in\alpha \\ h\in H_\alpha-\{b\}}} \lambda_h y_{mh} + \sum_{\substack{h\in\beta \\ h\in H_\beta-\{b\}}} \lambda_h y_{mh}, \quad m = 1, 2, \ldots, M;\ \ h = 1, 2, \ldots, H;$$

$$\sum\nolimits_{h\in\alpha} \lambda_h = Z_\alpha \tag{5}$$

$$\sum\nolimits_{h\in\beta} \lambda_h = Z_\beta$$

$$Z_\alpha, Z_\beta = 0 \text{ or } 1,\ Z_\alpha + Z_\beta = 1;$$

$$\lambda_1,\ldots,\lambda_H \geq 0;\ \theta_k \text{ without limitation on positivity and negativity}$$

$$a \in J_n\ ;\ b \in J_e\ ;\ h \in H - \{b\}$$

J_n is the set of non-SEDMUs; J_e is the set of SEDMUs.

In the above equation, Z_α and Z_β are grouping variables, defined as either 1 or 0. Let $Z_\alpha = 1$ and $Z_\beta = 0$; then, the efficiency value θ_α for the non-SE DMU_k in the α system can be obtained. Let $Z_\beta = 1$ and $Z_\alpha = 0$; then, the efficiency θ_β for the non-SE DMU_k in the β system can be obtained. The system-ranking-efficiency for the non-SEDMUs should thus be reassessed:

$$\Omega_{b,s} = \frac{\sum_{a \in J_n} \partial_{a,s}}{\widetilde{n}_s} \quad s \in \alpha, \beta \tag{6}$$

$\widetilde{n}_s$ is the number of non-SE DMUs of s-system;
b refers to the assessed SE DMUs;
$\partial_{a,s}$ denotes all the non-SE DMUs of s-system;
$\Omega_{b,s}$ is the efficiency of the SE DMUs of s-system.

In general, if the DMUs in an industry are homogeneous, the envelope for the input-oriented model will be concave downward. However, when non-homogeneous systems exist in an industry, the existence of non-homogeneous DMUs will cause the envelope for the whole industry to be neither concave nor downward. The system model can be considered an efficiency assessment method for such industrial exception. Thus, before using the non-homogeneous system model, the Mann-Whitney test can be applied to determine whether there is a significant difference in the production frontiers of the two systems. The present study establishes the models and deductions and then carries out empirical analysis on data from the Taiwan securities industry.

3. Analysis of Empirical Results

The current research uses Taiwan security companies as samples. Taiwanese authorities classify the securities industry into integrated securities firms and securities brokerage firms. Because these two types of firms differ in their scope and nature of business, this work uses the above procedural framework to carry out an efficiency ranking analysis for the Taiwan securities industry so as to resolve the problems of possible non-homogeneity and multiple efficiency DMUs in the industry. Annual data from the 2006 financial statement of the Taiwan securities industry are used. The sample includes 37 integrated securities firms and 34 securities brokerage firms. The input variables are fixed assets and number of employees, whereas the output variables are net operating income and non-operating income.

Table 1 Pearson Correlation Coefficients of the Variables

	Fixed assets	No. of employees	Operating income	Non-operating income
Fixed assets	1	0.909183	0.920696585	0.88752243
No. of employees	0.90918302	1	0.959050181	0.87549138
Operating income	0.92069658	0.9590502	1	0.90307068
Non-operating income	0.88752243	0.8754914	0.903070682	1

Source: Authors' calculation.

3.1 First Stage: Test whether Integrated Securities Firms and Securities Brokerage Firms Differ in Their Efficiency Boundaries

Integrated securities firms and securities brokerage firms differ in their business and operational structure. Thus, it is inappropriate to assess the efficiency of all the DMUs together. To determine whether the two types of securities firms have the same systems frontier, the study first estimates the efficiency value by applying the BCC model in the two systems according to type of securities firm. Table 2 shows the results. In System 1, the mean efficiency value for DMUs is 0.687754, the minimum efficiency value is 0.232419, and there are 8 efficiency DMUs. In System 2, the mean efficiency value for DMUs is 0.666141, and the minimum efficiency value is 0.093141, and; there are 10 efficiency DMUs. A comparison of the estimated efficiency values for between the two systems indicates that System 1 has better efficiency and less variability than System 2. Multiple efficiency DMUs exist in both systems, which presents difficulties in ranking all the DMUs.

Table 2 Efficiency Values for Integrated Securities Firms and Securities Brokerage Firms According to the BCC Model for Two Systems

System 1			System 2		
Integrated securities firms	**Score**	**Rank**	**Securities brokerage firms**	**Score**	**Rank**
DMU1	0.3332741	34	DMU38	0.9760902	11
DMU2	1	1	DMU39	1	1
DMU3	0.7227722	18	DMU40	0.4093511	27
DMU4	1	1	DMU41	0.5284319	22
DMU5	0.6908611	20	DMU42	0.4865522	24
DMU6	0.2324192	37	DMU43	0.8618038	13
DMU7	0.4695974	29	DMU44	1	1
DMU8	0.5047611	26	DMU45	0.0931407	34
DMU9	0.6539993	21	DMU46	1	1
DMU10	1	1	DMU47	1	1
DMU11	0.3888417	31	DMU48	0.468636	25
DMU12	0.89192	12	DMU49	1	1
DMU13	0.6221485	22	DMU50	1	1
DMU14	0.9790429	9	DMU51	1	1
DMU15	0.5917466	23	DMU52	0.7792786	15
DMU16	0.8401798	14	DMU53	0.1638941	33
DMU17	1	1	DMU54	0.562982	20
DMU18	0.7686473	15	DMU55	1	1
DMU19	1	1	DMU56	0.5839055	19
DMU20	0.4596174	30	DMU57	1	1
DMU21	0.7424602	17	DMU58	0.5457803	21
DMU22	0.5182091	25	DMU59	0.2136898	32
DMU23	0.926161	11	DMU60	0.2433533	31
DMU24	0.3441504	33	DMU61	0.792721	14
DMU25	0.3762417	32	DMU62	0.7053779	17
DMU26	0.4940093	28	DMU63	0.5017467	23
DMU27	0.7135796	19	DMU64	0.7532663	16
DMU28	1	1	DMU65	0.3745749	28
DMU29	0.2776062	36	DMU66	0.2721064	30
DMU30	0.3311202	35	DMU67	0.4417035	26
DMU31	0.5242839	24	DMU68	0.2933393	29
DMU32	0.50126	27	DMU69	0.6242118	18
DMU33	0.7501242	16	DMU70	1	1
DMU34	0.8533053	13	DMU71	0.9728511	12
DMU35	1	1			
DMU36	0.9445455	10			
DMU37	1	1			
MEAN	0.687754			0.666141	
MIN	0.232419			0.093141	

Source: Authors' calculation.

Further, when testing whether the two systems have the same boundary, it is necessary to determine the efficiency DMUs under the two systems, and then combine the efficiency DMUs for the two systems to estimate the efficiency value. The above BCC modeling results indicate that the two systems have a total of 18 efficiency DMUs. Table 3 shows there estimated efficiency values for the 18 DMUs. Of the 18 DMUs, 11 are efficiency DMUs; 8 of them belong to System 1, and only 3 are under System 2. By applying the Mann-Whitney test, the efficiency rankings for the DMUs under the two systems are shown to be significantly different (Table 4).

Table 3 Re-Estimated Efficiency Values for Each Efficiency DMU in the Two Systems

System	DMUs	Score	Rank
System 1	DMU2	1	1
	DMU4	1	1
	DMU10	1	1
	DMU17	1	1
	DMU19	1	1
	DMU28	1	1
	DMU35	1	1
	DMU37	1	1
System 2	DMU39	1	1
	DMU44	0.6570703	16
	DMU46	0.8352723	15
	DMU47	0.4374264	18
	DMU49	0.8546773	14
	DMU50	0.9996212	12
	DMU51	0.5858745	17
	DMU55	1	1
	DMU57	0.8949793	13
	DMU70	1	1

Source: Authors' calculation.

Table 4 Test Results on the Difference in Boundaries between the Two Systems

	Mann-Whitney U	Wilcoxon W	*Z*-value	*P*-value
Statistic	9.0	54.00	-3.164	0.002

Source: Authors' calculation.

3.2 Second Stage: Use the System-BCC Model to Estimate the Efficiency Values

The results from the first stage of the test prove that non-homogeneity exists in the Taiwan securities industry and that securities brokerage firms and integrated securities firms have different efficiency boundaries. Hence, it is necessary to apply the system-BCC model to estimate the efficiency values. Table 5 shows the estimation results. The securities firms are reviewed as two systems according to their business type. In System 1, the mean efficiency value for DMUs is 0.6858, the minimum efficiency value is 0.25096, and there are 8 efficiency DMUs. In System 2, the mean efficiency value for DMUs is 0.5534, the minimum efficiency value is 0.29916, and there are 3 efficiency DMUs. The mean efficiency value for all DMUs is 0.622396, and there are 11 efficiency DMUs. The performance of securities brokerage firms (SYS2) is thus relatively poor.

A comparison of the system-BCC modeling results with the estimation results obtained with by the BCC model after classification (as shown in Table 6) shows that the latter is unable to differentiate the efficiency performance between the two types of firms (meaningless due to different bases), whereas the former is able to show that the efficiency performance of integrated securities firms is better than that of securities brokerage firms. Further, the system-BCC model has greater influence on the efficiency estimation results for securities brokerage firms. The mean efficiency value decreased from 0.6661 to 0.5534, and the minimum efficiency from 0.2966 to 0.0931. The number of DMUs changed from 10 to 3.

Table 5 System-BCC Model Estimation Results

DMU	Score	Rank	System	DMU	Score	Rank	System
U1	0.3332741	58	1	DMU38	0.9113791	19	2
DMU2	1	1	1	DMU39	1	1	2
DMU3	0.7227722	29	1	DMU40	0.3182385	60	2
DMU4	1	1	1	DMU41	0.3849775	54	2
DMU5	0.6709156	31	1	DMU42	0.3426054	57	2
DMU6	0.2324192	66	1	DMU43	0.5199204	40	2
DMU7	0.4695974	47	1	DMU44	0.6570703	32	2
DMU8	0.5047611	43	1	DMU45	0.0931407	71	2
DMU9	0.6539993	33	1	DMU46	0.8352723	24	2
DMU10	1	1	1	DMU47	0.4374264	49	2
DMU11	0.3888417	52	1	DMU48	0.4065722	51	2
DMU12	0.89192	21	1	DMU49	0.9173994	18	2
DMU13	0.6221485	35	1	DMU50	0.9996212	13	2
DMU14	0.9790429	14	1	DMU51	0.9999852	12	2
DMU15	0.5917466	36	1	DMU52	0.5147662	42	2
DMU16	0.8401798	23	1	DMU53	0.145145	70	2
DMU17	1	1	1	DMU54	0.3086644	61	2
DMU18	0.7686473	26	1	DMU55	1	1	2
DMU19	1	1	1	DMU56	0.5242093	39	2
DMU20	0.4075308	50	1	DMU57	0.8949793	20	2
DMU21	0.7424602	28	1	DMU58	0.3864207	53	2
DMU22	0.5182091	41	1	DMU59	0.189158	68	2
DMU23	0.926161	17	1	DMU60	0.1729046	69	2
DMU24	0.3441504	56	1	DMU61	0.792721	25	2
DMU25	0.3762417	55	1	DMU62	0.5856129	37	2
DMU26	0.4940093	46	1	DMU63	0.4565677	48	2
DMU27	0.7135796	30	1	DMU64	0.4962634	45	2
DMU28	1	1	1	DMU65	0.1893883	67	2
DMU29	0.2776062	62	1	DMU66	0.2513425	63	2
DMU30	0.3311202	59	1	DMU67	0.2367948	65	2
DMU31	0.5242839	38	1	DMU68	0.2497078	64	2
DMU32	0.50126	44	1	DMU69	0.6242118	34	2
DMU33	0.7501242	27	1	DMU70	1	1	2
DMU34	0.8533053	22	1	DMU71	0.9728511	15	2
DMU35	1	1	1				
DMU36	0.9445455	16	1				
DMU37	1	1	1				
MEAN				0.622396			
S.D.				0.282937			
MAX				1			
MIN				0.09314			

Source: Authors' calculation.

Table 6 Comparison of the Estimated Efficiency Values between the System-BCC and the BCC Model

System-BCC model					
	Mean	S.D.	Max	Min	No. of efficiency DMUs
System 1	0.6858	0.2509	1	0.2324	8
System 2	0.5534	0.2991	1	0.0931	3

BCC model					
	Mean	S.D.	Max	Min	No. of efficiency DMUs
System 1	0.6877	0.2495	1	0.2324	8
System 2	0.6661	0.2966	1	0.0931	10

Source: Authors' calculation.

3.3 Third Stage: Use the System-Ranking-Efficiency Model to Resolve the Ranking Problem for Efficiency DMUs

The above estimation results obtained with the system-BCC model indicate that the use of the system-BCC model can solve the non-homogeneity problem that exists among companies in the industry. It is inappropriate to estimate the efficiency value for all DMUs together. Nonetheless, if the values for the two types of firms are estimated separately, the complete aspect for the industry cannot be obtained. However, the ranking problem with multiple efficiency DMUs remains. Therefore, the system-ranking-efficiency model must be applied in the last stage.

The theoretical background for DEA is based on a comparison of the distance of each DMU from the boundary. The boundary is an important factor. The DMUs on the boundary are the efficiency DMUs. The system-ranking-efficiency model starts with the concept of boundary change, considering the efficiency DMUs with the greatest influence on the boundary as the most important and, thus, as the ones that should have the highest efficiency ranking. To rank efficiency DMUs, the present study applied the system-ranking-efficiency model (as shown in Table 7), which successfully accomplished the task.

Table 7 Ranking Analysis for All Efficiency DMUs Based on the System-Ranking-Efficiency Model

DMU	Efficiency	Rank	System
DMU 2	0.572608	2	1
DMU 4	0.554008	8	1
DMU 10	0.569961	11	1
DMU 17	0.553571	10	1
DMU 19	0.571878	3	1
DMU 28	0.627562	1	1
DMU 35	0.560108	5	1
DMU 37	0.554282	6	2
DMU 39	0.554038	7	2
DMU 55	0.553637	9	2
DMU 70	0.562315	4	2

Source: Authors' calculation.

3.4 Analysis of Boundary DMUs

The system-ranking-efficiency model shows the effect of efficiency DMUs on the boundary type. After being deleted, the efficiency DMUs with the most influence on the boundary type have better efficiency rankings. The system-BCC model is based on the boundary concept used to estimate the efficiency values; however, its efficiency DMUs are a reference set; that is, the model selects the efficiency DMUs that are closest to the assessment units as a reference set. The number of times a DMU is referenced represents the importance of that DMU. Nevertheless, a DMU that is more frequently referenced does not necessarily have greater influence on the boundary type. To compare the boundaries between the system-BCC and the system-ranking-efficiency model, Table 8 presents the number of times the efficiency DMUs in the system-BCC model were referenced. Among all the DMUs, the most referenced was DMU28, followed by DMU19; these two were referenced over 30 times. DMU2 and DMU10 were the third and fourth most referenced, respectively, both more than 20 times. The other DMUs were referenced less than 10 times. DMU39 was referenced only once. A comparison with the data in Table 7 shows that both models ranked DMU28 as the most important. The second most important DMU in the system-ranking-efficiency model was DMU2; in the system-BCC model, it was DMU19. The third most important DMU was DMU19 in the system-ranking-efficiency model, compared with DMU2 in the system-BCC model. The most significant difference in ranking between the two models is found in DMU10, which was ranked 11th by the system-ranking-efficiency model and 4th by the system-BCC model. Further, DMU17 was ranked 10th by the system-ranking-efficiency model and 6th by the system-BCC model. Thus, the system-BCC model and the system-ranking-efficiency model differ in their ranking of the importance of DMUs. Therefore, the system-ranking-efficiency model is necessary.

Table 8 Number of Times the Efficiency DMUs Were Referenced in the System-BCC Model

DMU	2	4	10	17	19	28	35	37	39	55	70
System	1	1	1	1	1	1	1	2	2	2	2
No. of times the DMU was referenced	23	5	22	7	33	38	10	2	1	2	5

Source: Authors' calculation.

4. Conclusion

When the characteristic of non-homogeneity exists among the DMUs in an industry, it is inappropriate to estimate the efficiency value for all DMUs simultaneously. The use of an efficiency assessment method for a non-homogeneous system makes it possible not only to evaluate the efficiency for each DMU but also to compare the efficiency value for each system through each DMU. The proposed method can thus be applied to empirical analysis in various industries. The present study aimed to develop a system-ranking-efficiency model to solve the problems of non-homogeneity and efficiency ranking for DMUs.

The test results show that integrated securities firms perform better than securities brokerage firms in Taiwan. The system-ranking-efficiency model has more influence on securities brokerage firms and successfully solves the problem regarding multiple efficiency DMUs. Obtaining a complete efficiency analysis and ranking for Taiwan securities firms will increase our understanding of the benchmark for individual securities firms and promote an improvement in efficiency.

Further, our study results support the two features of the system-ranking-efficiency model. First, estimation by using the BCC model after classification fails to compare the efficiency performance between the two types of firms (meaningless due to different bases). On the other hand, the system-ranking-efficiency model is capable not only of assessing the efficiency for each DMU but also of comparing the efficiency value for each system through each DMU. Second, the boundary in the theoretical basis for DEA plays an important role. The DMUs on the boundary are the efficiency DMUs. The system-ranking-efficiency model in this study starts with the concept of boundary change, considering the efficiency DMUs with the greatest influence on the boundary as the most important DMU and, thus, as the ones that should have the highest efficiency ranking. After being deleted, the efficiency DMUs with more influence on the boundary type have better efficiency rankings.

Other DEA models for estimating the efficiency value are also based on the boundary concept. However, they use efficiency DMUs as a reference set; that is, the models select the efficiency DMUs that are closest to the assessment units as a reference set. The number of times a DMU is referenced represents the importance of that DMU. Nevertheless, a DMU that is more frequently referenced does not necessarily have greater influence on the boundary type. The system-ranking-efficiency model provides another assessment method, the necessity of which has been shown.

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