Endogenous Growth Effects of Environmental Policies

Summary: To analyse the impact of the environmental policies, we start by reviewing the literature on the environment, technological knowledge and economic growth. Then, we build a general equilibrium endogenous growth model where final goods are produced either in the skilled-labour intensive Clean sector or in the unskilled-labour intensive Unclean sector. By solving numerically transitional dynamics towards the unique and stable steady state, we observe that environmental policies encourage scale-invariant technological-knowledge bias. This, in turn, promotes environmental quality, the skill premium and economic growth. Moreover, the impact of population growth on the steady-state growth rate is higher under strong households’ environmental conscientiousness with future generations.

Key words: (Un)Clean sector, Environmental policies, Environmental conscientiousness, Growth, Wages.

JEL: C63, J31, O13, O31, Q55, Q58.

Several authors have stressed the tension between growth and the environment, showing that environmental degradation can create an endogenous limit to growth (Nancy Stokey 1998; Charles Jones 2009). Other authors have stressed that environment-friendly innovations do not stop sustainable long-run growth (Philippe Aghion and Peter Howitt 1998, Ch. 5).

In this context, it was not surprising that the Kyoto Protocol, including the Doha Amendment proposed and adopted in 2012, had come to promote environmental policies directed to generate clean technologies. However, by assuming exogeneity, the dominant literature on environmental policies has given little attention to technological knowledge: William Nordhaus (2007) proposes limited and gradual interventions, which reduce long-run growth; Nicholas Stern (2009) recommends extensive, permanent and immediate interventions, which involve relevant economic cost.

We contribute to this literature and present more optimistic results, by developing a dynamic general equilibrium endogenous growth model. In our model, final goods are produced either in the Clean sector or in the Unclean sector, and they use labour and intermediate goods, which in turn embody innovative designs under monopolistic competition. The former sector also uses Environmental Goods and Services (EG&S) provided by the government (e.g., A. Lans Bovenberg and Sjak Smulders 1996). Thus, due to EG&S, fiscal policy has an effect on growth (e.g., Ro-
Numerical calculations describing dynamic equilibrium towards the stable and unique steady state show that relevant and permanent environmental policies affect the technological knowledge, which, in turn, affects positively: (i) the environmental quality; (ii) the relative demand for labour and thus the skill-premium – in line with the path in developed and developing countries, since the 1980s (e.g., Elias Dinopoulos and Paul Segerstrom 1999; Daron Acemoglu 2009, Ch. 15); and (iii) the economic growth rate (e.g., Acemoglu 2009, Part IV). Additionally, EG&S incites an immediate-level effect, whereas subsidies only affect the technological-knowledge bias.

Moreover, we also take into account the growing concern of the existing population with future generations, reflecting a higher environmental conscientiousness, and we find that the impact of population growth on the steady-state growth rate is higher in this setting.

The paper is organised as follows. In Section 1 we review the related literature. In Section 2 we present the model. In Section 3 we derive the steady-state equilibrium. In Section 4 we analyse the effects of governmental intervention. In Section 5 we conclude with some remarks.

1. Literature on the Environment, Technological Knowledge and Economic Growth

The threat of climate change produced by the growing accumulation of greenhouse gases (GHG) in the atmosphere, largely due to the growing consumption of fossil fuels, has led to an increasing amount of research on climate change policy analysis. A large part of the discussion is focused on the effect of various policies in the development of alternative and more environmentally friendly productive modes. Thus, the Section proceeds by presenting the seminal theoretical literature (Subsection 1.1) and empirical literature (Subsection 1.2).

1.1 Theoretical Literature

Technological knowledge has become a central focus in environmental policy, since innovations can lower the cost and level of GHG. European Union programs on technological knowledge, such as the Renewable Energy White Paper and SAVE on energy efficiency, aim to stimulate environmentally friendly innovations. These innovations yield a double dividend: stimulate economic growth and generate fewer emissions (e.g., Herman Vollebergh and Claudia Kemfert 2005). However, for a long time, technological knowledge has been considered a black box in economics.
Rather than endogenous technological knowledge, the usual approach has been to use neoclassical growth models with exogenous technological knowledge. In particular, these models explore the implications of the exogenous technological knowledge path on factor prices, factor use, production and growth. Hence, most of the literature on environmental policy and growth has assumed exogenous technological knowledge, and thus environmental policy exerts only temporary effects on growth.

In the 1990s, new theoretical studies produced revisions of the neoclassical growth models allowing for technological knowledge endogeneity (e.g., Paul Romer 1990; Gene Grossman and Elhanan Helpman 1991; Aghion and Howitt 1992). The new research proposed well-articulated general equilibrium growth models based on R&D and sought to explore the role of technological knowledge in the growth process. In these new theoretical works, technological knowledge is determined by private and public sectors within the economic system, since economic agents can devote resources to R&D activity, and it is assumed that technological knowledge has features of public good: it is non-rival and only partially non-excludable (through patents).

Through endogeneity it becomes possible to analyse a wider scope of connections between environmental policy and technological knowledge. Indeed, the production possibilities with endogenous technological knowledge depend not only on time, but also on past, present and/or future expected prices and policies. For example, over longer time horizons, the models incorporating induced technological knowledge may project total costs of abatement substantially lower than those reported by conventional models with exogenous technological knowledge (Andreas Löeschel 2002).

According to Adam Jaffe, Newell, and Robert Stavins (2005), there are two interacting market failures related to R&D and environmental pollution. The public good nature of R&D requires subsidies to compensate for the difference between the social and private return of an investment. Negative externalities associated with the deterioration of environmental quality require corrective instruments such as taxes to restrict pollution. The role of environmental technology policy as the central point of the two interacting market failures is usually discussed, but the possibility of production with cleaner technologies is ignored.

In a model without technological-knowledge progress, pollution, as an externality from production, decreases the utility of the individuals implying that the optimal path for the economy is the convergence to a steady state with constant capital, production and consumption. With exogenous (e.g., Stokey 1998) or endogenous (e.g., Bovenberg and Smulders 1995) technological-knowledge progress, the possibility of the optimal trajectory to grow positively even in steady state is recovered.

Endogenous technological knowledge progress, together with the implications for sustainability, has shifted attention to the link between environmental policy and the direction of technological knowledge (e.g., Acemoglu et al. 2012). Indeed, despite the complexity introduced by considering the environment in endogenous growth models, a new growth literature has emerged with new insights on the environment-growth relation (e.g., André Grimaud and Luc Rougé 2003, 2008).
Concerning the relationship between economic growth and environmental policy, the work of Bovenberg and Smulders (1995, 1996) should be stressed. In the 1995 study, they offer a two-sector endogenous growth model where economic activity relies on the extractive use of the natural environment, modeled as a stock of natural capital. They explore the conditions under which physical output growth is sustainable and compatible with a stable quality of the natural environment. In contrast to most neoclassical models, where reproducible factors exhibit diminishing returns, they allow for constant returns to scale so that long-run growth is endogenously driven by preferences, technologies and policy.

Bovenberg and Smulders (1995) modeled a new technological knowledge that enables production to occur in a less polluting way and renewable resources to be used more efficiently. They argued two reasons for government intervention: pollution-diminishing knowledge and environmental quality. They found that, on an optimal balanced growth path, revenues from pollution taxes exceed public expenditures on the development of pollution-diminishing technology. They also analyse how a more ambitious environmental policy affects long-run equilibrium.

Bovenberg and Smulders (1996) explore the link between a tighter environmental policy and economic growth. They show that environmental policy exerts crucial impact on the long-run growth by inducing major technological-knowledge advances in abatement (environmentally friendly) technologies. Considering the environment a public consumption good, they also found that a reduced pollution level harms the productivity of man-made factors, depressing growth. If the environment acts as a public productive input, the enhanced environment quality improves productivity, offsetting the adverse growth effect of lower pollution.

Based on the quality-upgrading model of Aghion and Howitt (1992), Grimaud and Rougé (2003) showed that there are economic policy tools that allow the implementation of a balanced optimal growth path and they computed the precise levels of those tools. Francesco Ricci (2007) constructed a model with a continuum of goods differentiated in pollution intensity and concluded that the environmental taxation has a negative effect on a “green crowding-out effect”. Under a vintage model of production, Rob Hart (2008) showed that environmental taxes enhance growth and, with some restrictions, improve the environment. Minoru Nakada (2004) examined the impact of taxation on the profits and showed that environmental taxation has a positive impact on growth. Later, Nakada (2010) concluded that environmental tax, together with an income tax cut or a profits tax reduction, increases the growth rate.

Christian Groth and Poul Schou (2007) found that neither a tax on interest income nor a subsidy to capital accumulation affect the long-run growth rate; yet, policies directed towards the returns to resource protection affect growth. That is, resources taxes are rather of value for long-run growth, whereas traditional capital taxes and subsidies only affect levels. Don Fullerton and Seung-Rae Kim (2008) revealed that, relying on the absorption and the regenerative capacities of the environment, a higher pollution tax may increase growth.

Grimaud and Rougé (2008) verified that an environmental policy has two main effects: it postpones the resource extraction and with it the polluting emissions level, and it reallocates research efforts, reducing the grey research in benefit of the
green one. Pietro Peretto (2009) found that an energy tax has no effect on steady-state growth, though it has crucial transitional effects. The tax reallocates labour from energy to manufacturing inducing an increase in the aggregate R&D labour. However, this productivity growth acceleration is temporary since the manufacturing increase attracts entry, which creates dispersion of R&D resources across firms, offsetting the increase in aggregate R&D.

Other studies on the link between environmental policy and economic growth could be referenced; for example: Bovenberg and Ruud A. Mooij (1997), Frank Hettich (1998) and Alfred Greiner (2005), who consider a pollution tax and a distorting income tax; Bob Zwaan et al. (2002), who analyse the effect of environmental policies on technological knowledge; Reyer Gerlagh, Snorre Kverndokk, and Knut Rosendahl (2009), who stress that using R&D subsidies would enable lower carbon taxes.

Turning now to the issue of modeling, we start by referencing Löeschel’s (2002) survey, which reveals the approaches about technological knowledge and energy-environment models. This survey shows that technological-knowledge processes are described by a substantial uncertainty due to the arrival time and performance of new technologies. Vollebergh and Kemfert (2005), in turn, present the state-of-the-art discussion of recent theoretical and empirical advances on technological knowledge and the environment.

Kenneth Gillingham, Newell, and William Pizer (2008) provide an overview of the approaches used to model technological knowledge. They show that no single approach appears to dominate and, relying on the purpose of the analysis, different approaches may be preferred. They conclude that the simplest approach is the exogenous one assuming Hicks-neutral productivity. However, it does not capture the potential for technological-knowledge change to proceed in an energy-saving manner. An easy way to overcome such weakness is to include an energy-efficiency improvement parameter, which increases the energy efficiency of the economy by some exogenous amount each year.

Another form of exogenous technological knowledge is the backstop technology. Backstop technologies are carbon-free energy sources that may be already known, but not yet widely traded. It is usually assumed that they are available in unlimited supply at a fixed and relatively high marginal cost (the backstop price) that reflects the associated R&D costs. When they mature, costs fall with technological-knowledge progress: e.g., advanced solar power, nuclear fusion and renewable transportation fuels.

Concerning the endogenous technological-knowledge modeling, there are three major approaches: the (neoclassical) direct price-induced, R&D-induced and learning-induced. The former suggests that changes in relative prices can spur innovation to cut the use of the more expensive input (e.g., energy); thus, R&D activities result from market conditions. The following allows for R&D investment to affect the direction and rate of technological knowledge; this modeling is associated with spillover effects (i.e., investments that benefit the investor and others). The latter relies on learning concepts; here, the productive costs and/or abatement fall with the accumulated production experience; hence, historical events affect future possibilities (e.g., Vernon Ruttan 1997).
Gillingham, Newell, and Pizer (2008) found that, although there is no single best approach to make technological knowledge endogenous, some methods appear better suited than others. For instance, approaches based on feedback through R&D activities have been fruitfully developed in more highly aggregated optimization models. In turn, learning-induced approaches have been mostly used in disaggregated energy technology and systems models. Furthermore, R&D activities typically use top-down models that focus on the economy as a whole rather than on detailed descriptions of specific sectors.

Learning-induced approaches typically use bottom-up models that are technology-oriented, focusing on the description and the economic performance of the various technologies. These models represent technological knowledge through the replacement of one technology by another, due to the better performance of the latter. Both R&D and learning curves respond to changes in relative prices and hence the rate and direction of technological knowledge are now sensitive to price incentives.

R&D-induced technological knowledge is another approach used to make technological knowledge endogenous. The above quoted endogenous growth literature (e.g., Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992) stresses the importance of the knowledge capital, that captures all information, skills, ideas and experience in economic growth models. In these models, innovation is treated as a result of explicit profit-maximizing investment in R&D, which attributes private and public properties to technological knowledge, providing appropriation of new technological knowledge and spillovers (or positive technological-knowledge externalities).

Regarding the endogenous directed technological knowledge in models of growth and the environment, Acemoglu (2002) and Nordhaus (2002) should also be referenced. It is, however, Acemoglu et al. (2012) who develop the more general framework to perform comparative analysis for the effects of different types of policies on innovation, growth and environmental resources. These authors describe the structure of optimal regulation and study the implications of dirty inputs using exhaustible resources. They show, under substitutability between inputs, that: (i) sustainable growth can be obtained with temporary taxation of dirty activities (R&D and production); and (ii) using exhaustible resources as an input in dirty production helps to increase R&D activity in the clean sector. They also show that the delay in intervention involves medium-run growth costs.

Gerlagh and Wietze Lise (2005) develop the partial equilibrium model Deme-ter-2E for energy supply and demand with endogenous technological knowledge induced by R&D and learning-by-doing. This bottom-up model follows from a carbon-based and a non-carbon-based technology. It allows for energy source substitution, which is crucial for abatement of carbon dioxide emissions, as required by the Kyoto Protocol. Transition from one energy source to the other is endogenous and the policy is represented by a carbon tax. The model produces a transition from fossil fuel to carbon-free energy sources within the next two centuries, with a pattern that follows the gradual diffusion S-curve.

Three recent theoretical studies should also be mentioned. John Hassler and Per Krusell (2012) develop a dynamic stochastic general-equilibrium model with a
continuum of regions that integrates the climate; in particular, the authors show that only taxes on oil producers can improve the climate. Mikhail Golosov et al. (2014) propose a dynamic stochastic general-equilibrium model with an externality through climate change from using fossil energy, resulting that under quite plausible assumptions a marginal externality damage of emissions is proportional to current Gross Domestic Product. Finally, Acemoglu et al. (forthcoming) develop a microeconomic model of endogenous growth where clean and dirty technologies compete in production and innovation, showing that, if dirty technologies are more advanced to start with, the potential transition to clean technology can be difficult.

1.2 Empirical Literature

Concerning the empirical literature, the study by Ger Klaassen et al. (2005) focuses on cost-reducing innovation in wind turbines in three countries – Germany, Denmark and the UK. These authors study the innovation and diffusion mechanism using the two-factor learning curve that is a typical bottom-up approach on the progress and spread of new technologies. Some other works try to find how credible the learning curves are in describing endogenous energy technological advances (e.g., Haoran Pan and Jonathan Köhler 2007) and others still seek to study how learning curves for renewable energy technologies can be integrated into a dynamic programming model (e.g., Gurkan Kumbaroglu, Reinhard Madlener, and Mustafa Demirel 2006).

Pan and Köhler (2007), using an application to the UK wind power, showed that the logistic curve can better depict the bottom-up technological change than the learning curve approach. Kumbaroglu, Madlener, and Demirel (2006) evaluated the investment alternatives for the Turkish electricity industry assuming that the flexibility to delay irreversible investment expenditure can notably affect the diffusion of different power generation technologies.

Some other papers, such as Hans Gerbach and Till Requate (2004) analyse how the regulatory regime via emissions taxes or standards may affect the firms’ adoption of emissions abatement technology. Using panel data, Popp (2003) provides a study that checks the effects of introducing the tradable permit system for CO₂ emissions as part of the US Clean Air Act Amendments. Popp (2004) allowed for directed innovation in the energy sector and revealed, by a calibration exercise, that models ignoring the directed technological change effects can notably exacerbate the cost of environmental regulation. In turn, Popp (2006) looks at the experience of the US, Japan and Germany with respect to patents for CO₂ and NOₓ abatement technologies.

On the same empirical line, Johnstone, Hascic, and Popp (2010) examine innovations for renewable energy, using a panel data set comprised of 25 countries over 26 years. Their aim was to observe how the effectiveness of a wide variety of policy types, including feed-in-tariffs, production quotas and public R&D expenditures varies by technology.

There is also a very interesting study by Lise and Gideon Kruseman (2008) who analyse the effects of endogenous investment decisions on prices and on the environment in a liberalized electricity market. They use a game-theoretic recursive dynamic model – the dynLem – that is an extension of the Emelie model applied to
the European electricity market. It includes a static part (trade, capacity and environmental constraints) and a recursive dynamic part (the investment decision in the production capacity). For calibration, the authors use perfect competition, strategic competition (large firms exercising market power) and sequential strategic competition (Stackelberg market leader).

It should also be emphasized the recent empirical works of Benjamin Jones and Benjamin Olken (2010), Matteo Lanzafame (2014), Ayodele Odusola and Bata-tunde Abidoye (2015) and Aghion at al. (forthcoming). Jones and Olken (2010) analyse the trade effects and export performance of developing countries to climate change and conclude that warmer temperatures tend to dampen export performance of developing countries. Lanzafame (2014) investigates the effects of temperature and rainfall on economic growth also in Africa and the author finds evidence of both short- and long-run relationships between temperature and per capita income growth. Odusola and Abidoye (2015) examined the impact of climate change on economic growth in Africa and they found found a negative impact of climate change on economic growth. Finally, Aghion at al. (forthcoming) construct new firm-level panel data on auto industry innovation distinguishing between “dirty” and “clean” patents across 80 countries over several decades, and they find a sizable impact of carbon taxes on the direction of innovation in the automobile industry and further provide evidence that clean innovation has a self-perpetuating nature feeding on its past success.

2. The Model

The model developed and analysed in this section is more closely related to studies that address the environmental policy issue using endogenous growth models focusing on the direction of technological knowledge (e.g., Acemoglu 2002; Ricci 2007; Acemoglu et al. 2012).

2.1 Product and Factor Markets

Each perfectly competitive final good \( n \in [0, 1] \) is produced by the Unclean or the Clean sector. The former (latter) uses unskilled (skilled) intensive labour, \( L (H) \), and a continuum set of intermediate goods, \( j \in [0, J] (j \in ]J, 1[) \). Output of \( n \), \( Y_n \), at time \( t \) is:

\[
Y_n(t) = A \left\{ \int_0^1 \left( q^{k(j)} x_a(k,j,t) \right)^{1-a} d_j \right\} \left[ 1^{-n} L_n(t) \right]^a + \left[ \int_0^J \left( q^{k(j)} x_a(k,j,t) \right)^{1-a} d_j \right] \left[ n G H_a(t) \right]^a \right\} . \tag{1}
\]

\( A > 1 \) is the exogenous productivity level. In the Schumpeterian tradition, integrals denote the aid of intermediate goods: each \( j \) quantity, \( x \), is quality-adjusted; the quality upgrade is \( q > 1 \), and \( k \) is the top rung at \( t \). Parameter \( a \in ]0, 1[ \) is the labour share in production. In line with Bovenberg and Smulders (1996), EG&S provided by the government, \( G > 1 \), creates a productivity advantage of \( H \) over \( L \). Terms \((1-n)\) and \( n \) imply that \( L \) is more productive in goods indexed by smaller \( ns \) and vice-versa. As it will be clear below, sector optimal choice at \( t \) is reflected in the endogenous threshold final good \( \bar{n} \), where the switch from the production by \( L \) to \( H \) is advantageous.
Due to zero profit equilibrium by producers of $n \in [0, 1]$, the demand for the top quality of $j$ by the producer of $n$ is:

$$x_n(k, j, t) = \left[ \frac{p_n(t) A(1-\alpha)}{p(k, j, t)} \right]^{1/\alpha} q^{k(j, t)[1-(1-\alpha)/\alpha]} \begin{cases} (1-n) L_n(t), & \text{if } 0 < j \leq J \\ n G H_n(t), & \text{if } J < j \leq 1 \end{cases},$$

where $p_n$ and $p(j)$ are, respectively, the prices of $n$ and $j$. A higher: (i) $p_n$ increases the marginal revenue product of factors, encouraging firms to rent more $j$s; (ii) $m_n (= L_n, H_n)$ implies more labour to use $j$s, raising demand; and (iii) $p(j)$ implies lower demand, since the demand curve for $j$s is downward sloping. As we will see below, due to profit maximising by monopolist producers, $p(j)$ is independent of $j$. From (2) and (1), the supply of $n$ is:

$$Y_n(t) = \frac{p_n(t) (1-\alpha)}{p(j, t)} e^{(1-\alpha)} [n G H_n(t) Q_H(t) + (1-n) L_n(t) Q_L(t)],$$

where:

$$Q_L = \int_0^j q^{k(j, t)[1-(1-\alpha)/\alpha]} dj \quad \text{and} \quad Q_H = \int_j^1 q^{k(j, t)[1-(1-\alpha)/\alpha]} dj$$

are aggregate quality indexes, evaluating the technological knowledge in each range of intermediate goods, and $D \equiv Q_H/Q_L$ assesses the technological-knowledge bias; i.e., the relative productivity of the (Clean) technological knowledge used together with $H$.

We define the aggregate output, i.e., the composite final good, as:

$$Y(t) = \int_0^1 p_n(t) Y_n(t) dn = \exp \left[ \int_0^1 \ln p_n(t) dn \right] \exp \left[ \int_0^1 \ln Y_n(t) dn \right] = \exp \left[ \int_0^1 \ln Y_n(t) dn \right],$$

where $p_n(t)$ is the $n$ price and we normalise the price of $Y$ at each time $t$ to one (numeraire): $\exp \int_0^1 \ln p_n(t) dn = 1$. Resources, $Y$, that are not consumed, $C$, are used in the production of intermediate goods, $X$, and in the R&D sector, $R$; i.e., $Y = X + R + C$.

Economic viability of either substitute type of technology in (1) relies on: (i) prices of labour and the relative productivity, $G$; and (ii) prices of intermediate goods and the relative productivity. The prices of labour rely on the quantities $H$ and $L$. In relative terms, the productivity-adjusted quantity of $H$ in production at each $t$ is $\frac{Q_H}{L}$.

As for the prices of intermediate goods and productivity, they depend on complementarity with either labour type, on the technological knowledge embodied, and on the mark-up, which in turn relies on the elasticity of demand by the producers of final goods. These determinants are summed up in the aggregate quality indexes in (4), $Q_H$ and $Q_L$.

Just as in, for example, Acemoglu and Fabrizio Zilibotti (2001) and Oscar Afonso (2006, 2008), the resulting threshold final good $\Pi$, determining the exclusive use of the Clean (or $H$) technology in final goods $n > \Pi$ and of the Unclean (or $L$) technology for $n \leq \Pi$, follows from profit maximisation by firms and full-employment equilibrium in factor markets, given labour supply and technological
knowledge. It is a function of the determinants of economic viability of the two technologies (see the proof in the Appendix):

\[ \bar{n}(t) = \left\{ 1 + \left[ \frac{Q_H(t) G H(t)}{Q_L(t) L(t)} \right]^{1/\alpha} \right\}^{-1}, \]  

(6)

where the threshold final good \( \bar{n} \) is small (the number of Clean final goods is large) when \( D \) is highly biased, \( H \) and/or EG&S are large. The threshold final good can be related to prices since at \( \bar{n} \) both an Unclean \( L \)-technology firm and a Clean \( H \)-technology firm should break even. The index prices of final goods produced with \( H \) and \( L \)-technologies (respectively \( p_H \) and \( p_L \)) and the respective ratio are

\[
\begin{align*}
 p_L &= p_n (1 - n) = \exp(-\alpha) \bar{n}^{-\alpha} \\
 p_H &= p_n n^\alpha = \exp(-\alpha) (1 - \bar{n})^{-\alpha}
\end{align*}
\]  

and thus

\[ P(t) \equiv \frac{p_H(t)}{p_L(t)} = \left[ \frac{\bar{n}(t)}{1 - \bar{n}(t)} \right]^{\alpha}. \]  

(7)

From (7), small \( \bar{n} \) implies a small relative \( H \) final-goods price: the demand for each \( j \in [\mathcal{J}, 1] \) is low, which, as will be apparent below, affects R&D direction; i.e., \( \bar{n} \) influences the R&D direction through the price channel.

To sum up, when either the technology is highly Clean biased (i.e., \( D \equiv \frac{Q_H}{Q_L} \) is high) or there is a large relative supply of \( H \) (i.e., \( \frac{H}{L} \) is high) the fraction of final goods using the Clean technology is large and \( \bar{n} \) is small – see (6). This implies a low relative price of final goods produced with the Clean technology – see (7). In this case, by (2), the demand for Clean intermediate goods is low, which, as we will see below, due to the price channel discourages R&D aimed at improving their quality. Thus, labour levels affect the direction of R&D and thus \( D \) through the price channel. Indeed, the incentives to develop specific technologies are weaker when the prices of final goods produced with these technologies are lower due to the use of the more abundant labour type. This price channel shows up in various papers by, for example, Acemoglu (e.g., 2009), although always dominated by the market-size effect, which, as will be clear later on, is removed in our case.

The equilibrium aggregate resources devoted to intermediate goods, \( X(t) \equiv \int_0^1 \int_{\mathcal{J}} x_n(k, j, t) dj dn \), and the equilibrium aggregate output, \( Y \) in (5), are expressed as a function of current technological knowledge and labour levels; for example, \( Y \) is:

\[ Y(t) = \int_0^1 p_n(t) Y_n(t) dn = \exp(-1) A^{1/\alpha} \left[ \frac{1 - \alpha}{q (1 - s_y)} \right]^{1-\alpha/\alpha} \left[ \left( \frac{Q_H(t) L(t)}{Q_L(t)} \right)^{\alpha} + \left( \frac{G H(t)}{Q_n(t) L(t)} \right)^{\alpha} \right]^2. \]  

(8)

Equation (8) shows that the country’s growth rate is driven by the technological-knowledge progress and by labour growth. Due to (8), the price paid for labour unit, \( w_m (m = L, H) \), is equal to its marginal product and the resulting skill-premium, \( W \), is:
The relative wages $W$ is driven by the relative demand, which is affected by the technological-knowledge bias, $D$, by the relative productivity, $G$, and by the labour level structure, $\frac{H}{L}$. An increase in $G$ is a static benefit, see (8), which affects $W$ directly (level effect), see (9). The increase in $G$ also affects $W$ indirectly (dynamic effect): due to complementarity between inputs, the change in $G$ affects $D$ by the price channel during the transition phase (towards a new steady state). Indeed, (6), (7) and (9) are particularly useful in foreseeing the operation of the price channel from the levels of technological-knowledge and labour to the flow of resources used in R&D and to wage inequality. For example, when $\tilde{m}$ is high, profit opportunities in the production of intermediate goods used by the relative high-priced final goods produced with the Clean technology induce a direction of R&D in favour of high $D$ and thus in favour of high $W$.

2.2 Intermediate Goods and R&D Activities

Since $Y$ is the input in the production of $j \in [0, 1]$ and final goods are produced in perfect competition, the marginal cost of production of $j \in [0, 1]$ is 1. Moreover, considering that the government can pay an ad-valorem fraction, $s_x$, of each firm’s cost, $(1-s_x)$ is the after-subsidy marginal cost. The production of $j$ requires a start-up cost of researching a new design, which can only be recovered if profits at each date are positive for a certain time in the future. This is guaranteed by a domestic system of intellectual property rights, which protect the leader firm’s monopoly, while at the same time and at almost no cost, disseminating acquired technological knowledge to other firms. The profit-maximisation price of the firms yields the constant over $t$, across $j$ and for all $k$ mark-up $p(k,j,t) = p = \frac{1-s_x}{1-\alpha} > 1$, since $\alpha > s_x$.

Since the leader is the one legally allowed to produce the top quality, it will use pricing to wipe out sales of lower quality. Relying on whether $q(1-\alpha) > 1$ or $q(1-\alpha) < 1$, the leader will use either the monopoly pricing $p = \frac{1-s_x}{1-\alpha}$ or the limit pricing $p = q(1-s_x)$ to capture the entire market. Such as Grossman and Helpman (1991, Ch. 4), we assume that the limit pricing is binding. Thus, since the lowest price that the closest follower can charge without negative profits is $(1-s_x)$, the leader can capture the whole market by selling at a price slightly below the quality advantage over the closest follower.

We follow the Schumpeterian growth models, but we assume that leaders use some strategies to protect their economic rents by delaying the next successful R&D explored by a follower, and that further scale also entails other costs, which, together, enable us to abolish scale effects, as suggested by the main literature on scale effects since Jones (1995). The value of the leading-edge patent relies on the profit-yields accruing at each $t$ to the monopolist, and on the duration of the monopoly. The duration, in turn, relies on the probability of a new innovation, which creatively destroys the leading-edge design, and thus is at the heart of the R&D sector (e.g., Aghion and...
Howitt 1992). R&D outcomes are thus designs to improve indexes in (4) – e.g., Acemoglu 2009, Ch. 14. In line with, e.g., Afonso (2006, 2008), let \( pb(k, j, t) \) be the probability of successful innovation at \( t \) (Poisson arrival rate) in the next quality of \( j \), \( k(j, t) + 1 \), that complements the labour type \( m \) (\( m = L \) if \( 0 < j \leq J \) and \( m = H \) if \( J < j \leq 1 \)).

\[
pb(k, j, t) = y(k, j, t) \cdot \beta q^{k(j, t)} \cdot \zeta^{-1} q^{-a^{-1}k(j, t)} \cdot m(t)^{-1},
\]

where:

(i) \( y(k, j, t) \) is the flow of aggregate final good resources devoted to R&D in \( j \) at \( t \), which defines our framework as a lab-equipment model.

(ii) \( \beta q^{k(j, t)} \), \( \beta > 0 \), is the positive learning effect of accumulated public technological knowledge from past successful R&D in \( j \) at \( t \); \( \beta \) is the parameter on learning-to-past successful R&D, and a larger \( \beta \) means a better innovation capacity.

(iii) \( \zeta^{-1} q^{-a^{-1}k(j, t)} \), \( \zeta > 0 \), is the adverse effect in \( j \) at \( t \) caused by the increasing complexity of quality improvements. Hence, \( \zeta \) corresponds to the fixed cost of R&D.

(iv) \( m(t)^{-1} \) is the adverse market-size effect at \( t \), capturing the idea that the complexity of changing qualities of intermediate goods depends on the market size measured by \( m \). Hence, the exponent -1 measures the use of generic costs and rent protecting actions to remove the scale effects and thus to isolate the price channel (this will be clear below). The scale increasing costs are reflected here due to coordination among agents, processing of ideas, informational, organisational, marketing and transportation costs (e.g., Dinopoulous and Segerstrom 1999). Moreover, since the rents of leader firms rely on the market size and this term also includes leader firms’ strategies involving technical barriers (e.g., Dinopoulous and Constantinos Syropoulos 2007), which delay the next creative innovation to retain the respective economic rents, through this term we are also able to distinguish between R&D leading to creative destruction and leader firms’ strategies that slow down the creative destruction.

The incentives of follower firms to perform R&D relies on the expected monopoly profits flow, \( V(k, j, t) \), which relies on its duration, the interest rate, \( r \), and the profits at each \( t \), \( \Pi(k, j, t) \):

\[
\Pi(k, j, t) = \bar{m} m(t) (1 - s_{x,m})^{a^{-1}(1-\alpha)} (q - 1) \left[ \frac{p_m(t) A (1-\alpha)}{q} \right]^{a^{-1}} q^{k(j, t) a^{-1}(1-\alpha)},
\]

where \( \bar{m} = G \) for \( m = H \), \( \bar{m} = 1 \) for \( m = L \), and \( s_x \) can be \( m \)-specific. The resulting \( V \) is:

\[
V(k, j, t) = \frac{\Pi(k, j, t)}{r(t) + pb(k, j, t)}.
\]

That is, the expected income generated by successful R&D on rung \( k \), \( V r \), equals the profit flow, \( \Pi \), being paid out as dividends, minus the expected capital loss, \( V pb \), that will occur when \( k \) is replaced. The effective discount rate is thus \( r + pb \). Under free-entry R&D equilibrium, expected returns are equal to resources spent:
where $s_r$ is a governmental ad-valorem subsidy to R&D, which can be $m$-specific. Considering (10), (11) and (12), the equilibrium $m$-specific probability of a successful R&D, $p_{b_m}$, is:

$$p_{b_m}(t) = \frac{\beta}{\zeta} \left( \frac{1 - s_{m,s}}{1 - s_{m,r}} \right) \left( \frac{q - 1}{q} \right) \left[ \frac{p_{m}(t) A (1 - \alpha)}{1 - s_{m,s}} \right]^{\alpha - 1} \frac{\bar{m} - r(t)}{q^{\alpha - 1} (1 - \alpha)} - 1. \quad (14)$$

Thus, $p_{b_m}$ turns out to be independent of $j$ and $k$, due to the removal of scale effects. The positive effect of the quality rung on profits and on the learning effect is exactly offset by its effect on the complexity cost – see (10)-(ii), (10)-(iii) and (11) together. Scale effects could also arise through the market size, as has been debated in the R&D growth literature since Jones’ (1995) critique. To highlighting the price channel, the adverse effect of market size in (10)-(iv) is designed to offset the scale effect on profits in (11): computing $I_H - I_L$, the technological-knowledge bias $D$ is strongly induced by environmental policies under the price-channel mechanism.

From (10) and (14), the equilibrium aggregate resources devoted to R&D, $R(t) \equiv \int_0^t y(k, j, t) dj$, also is expressed as a function of current technological knowledge. However, the increased resources devoted to R&D as $Q_m$ rises does not lead to greater rates of successful R&D, but rather are needed to offset the difficulty of R&D as $Q_m$ increases. Since $p_{b_m}$ governs the speed of technological-knowledge progress, equilibrium can be translated into the path of $Q_m$ (Technology-curve):

$$\hat{Q}_m(t) = \left\{ \frac{\beta}{\zeta} \left( \frac{1 - s_{m,s}}{1 - s_{m,r}} \right) \left( \frac{q - 1}{q} \right) \left[ \frac{p_{m}(t) A (1 - \alpha)}{1 - s_{m,s}} \right]^{\alpha - 1} \frac{\bar{m} - r(t)}{q^{\alpha - 1} (1 - \alpha)} - 1 \right\} \left[ q^{\alpha - 1} (1 - \alpha) - 1 \right]. \quad (15)$$

### 2.3 Consumers

Infinitely-lived households inelastically supply $L$ or $H$, and maximize inter-temporal utility from consumption per capita $c \equiv C / (L + H)$:

$$U(t) = \int_0^\infty \left[ \frac{c(t)^{1-\theta} - 1}{1-\theta} \right] \exp(-\rho + e \eta) t dt, \quad (16)$$

where $-\rho + e \eta < 0$ in order to ensure that $U$ is bounded away from infinity if $c$ were constant over time; $\theta > 1$ is the relative risk aversion coefficient; $\rho > 0$ is the subjective discount rate; $\eta$ is the population, $L+H$, growth rate; and $e \in [0, 1]$ is the households’ level of concern with future generations, reflecting environmental conscientiousness. Thus, $e = 0$ defines the minimum degree of environmental conscientiousness, whereas the opposite limiting case $e = 1$ describes the case with perfect environmental conscientiousness.

A household faces the budget constraint:
\[ \dot{k}(t) = [r(t) - \eta]k(t) + w_m(t) m - c(t) - T(t), \] (17)

where \( k \) is the total household’s asset holdings, with return \( r \), in the form of ownership of leaders (and not in public debt, since, by assumption, the government budget is always balanced); (ii) \( T \) is a lump-sum tax on the household to finance EG&S and subsidies.

The (representative) household maximizes utility of per capita consumption (16), subject to the budget constraint (17) and to the standard no Ponzi games condition. The solution for the optimal path of household per capita consumption is (Euler curve):

\[ \frac{\dot{c}(t)}{c(t)} = \frac{r(t) - \rho - \eta (1 - e)}{\theta} \Rightarrow \frac{\dot{c}(t)}{C(t)} = \frac{\dot{c}(t)}{c(t)} + \eta = \frac{r(t) - \rho - \eta (1 - e - \theta)}{\theta}. \] (18)

In particular, from (18), \( c \) depends negatively on \( \eta \) if \( (1 - e) > 0 \) or is independent of \( \eta \) when \( e = 1 \). In turn, \( C \) relies positively on \( \eta \) since \( (1 - e - \theta) < 0 \). To sum up, the effect of population growth on consumption growth rate, \( \dot{c}/c \) or \( \dot{C}/C \), relies positively on the households’ environmental conscientiousness with future generations.

### 3. Steady-State Equilibrium

\( Q_L \) and \( Q_H \) must grow at the same rate since (i) \( Y \) has constant returns to scale in inputs, (ii) \( Y, X, R \) and \( C \) are multiples of \( Q_L \) and \( Q_H \), and (iii) in steady-state aggregates grow at the same rate. From (15), \( \hat{Q}_H = \hat{Q}_L \) if \( p_H/p_L = \left( \frac{1 - s_{x,m}}{1 - s_{x,m}} \right)^{\alpha} \left( \frac{1 - s_{r,m}}{1 - s_{r,m}} \right)^{\alpha} G^{-\alpha} \); since \( r \) is unique, the steady-state growth rate, \( g^* \), is thus also unique. Also, from (6) and (7),

\[ \frac{p_H}{p_L} = \left( \frac{D}{G} \right)^{\left( \frac{\alpha}{2} \right)}. \]

Consider, e.g., \( p_{b_H} > p_{b_L} \) implies\( \hat{Q}_H > \hat{Q}_L \) and, since \( \frac{p_H}{p_L} = \left( \frac{1 - s_{x,m}}{1 - s_{x,m}} \right)^{\alpha} \left( \frac{1 - s_{r,m}}{1 - s_{r,m}} \right)^{\alpha} G^{-\alpha} \). \( p_{b_H} > p_{b_L} \) implies that \( \hat{Q}_H > \hat{Q}_L \) and, since \( \frac{p_H}{p_L} = \left( \frac{1 - s_{x,m}}{1 - s_{x,m}} \right)^{\alpha} \left( \frac{1 - s_{r,m}}{1 - s_{r,m}} \right)^{\alpha} G^{-\alpha} \), which attenuates the rate at which \( D \) is rising. Thus, while \( \hat{Q}_H > \hat{Q}_L \), \( \hat{Q}_H - \hat{Q}_L \) is falling until it achieves the stable \( g^* \), where \( \hat{Q}_H = \hat{Q}_L \), which, by (15), also implies a stable \( r^* \):

\[ g^* = \hat{Q}_H = \hat{Q}_L = \hat{Y} = \hat{X} = \hat{R} = \hat{C} = \frac{r^* - \rho - \eta (1 - e - \theta)}{\theta} \Rightarrow \hat{P}_H = \hat{P}_L = \hat{n} = W^* = 0. \] (19)

Hence, by \( G, s_{x,m} \) and \( s_{r,m} \), government positively affects \( g^* \), by encouraging R&D: \( G \) and \( s_{x,m} \) boost profits (11) and \( s_{r,m} \) decreases the R&D cost, see (13). Moreover, the effect of population growth on the steady-state economic growth rate depends positively on the households’ environmental conscientiousness with future generations.
4. Government Intervention

As \( r \) is unique, (15) is used to analyse the effect on \( \bar{\eta} \) and \( W \) of the \( D \) path given by:

\[
\dot{D}(t) = \frac{\beta}{\zeta} \left( \frac{q-1}{q} \right) \left( A(1-\alpha) \right)^{1 \alpha} \exp(-\alpha) \\
\left\{ G \left( \frac{1-s_{x,H}}{1-s_{x,H}} \right) \right\}^{1 \alpha} \left[ 1 + \left( \frac{D(t) G H}{L} \right)^{1 \alpha} \right] \left( \frac{1-s_{r,H}}{1-s_{r,H}} \right) \left( \frac{1-s_{x,L}}{1-s_{x,L}} \right) \left[ 1 + \left( \frac{D(t) G H}{L} \right)^{1 \alpha} \right],
\]

using the 4th-order Runge-Kutta numerical method and the baseline values in Table 1.

Table 1 Baseline Parameters and Labour Levels

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Variables</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )</td>
<td>1.05</td>
<td>( \beta )</td>
<td>1.60</td>
<td>( \rho )</td>
<td>0.02</td>
<td>( A )</td>
<td>1.50</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.70</td>
<td>( \zeta )</td>
<td>4.00</td>
<td>( s_{s,m} ), ( s_{o,m} ), ( T )</td>
<td>0.00</td>
<td>( H(t=0) )</td>
<td>0.68</td>
</tr>
<tr>
<td>( q )</td>
<td>3.33</td>
<td>( \theta )</td>
<td>1.50</td>
<td>( \eta ), ( e )</td>
<td>0.00</td>
<td>( L(t=0) )</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note: Values are in line with our assumptions (\( G > 1 \), \( \beta > 0 \) and \( \zeta > 0 \)), Acemoglu (2009) and to calibrate \( g^* \) around 2.5% under (Scenario, Sc0) no governmental intervention.

Source: Authors’ assumptions, based on theoretical framework and on the literature; i.e., values are in line with our assumptions (\( G > 1 \), \( \beta > 0 \) and \( \zeta > 0 \)), Acemoglu (2009) and to calibrate \( g^* \) around 2.5% under (Scenario, Sc0) no governmental intervention.

Figures 1a, 1b and 1c below compare the baseline steady-state paths of \( D \), \( \bar{\eta} \) and \( W \) with those arising from a change at \( t = 0 \) where: Sc1, \( s_{x,H} = 0.2 \); Sc2, \( s_{r,H} = 0.2 \); Sc3, \( s_{x,H} = 0.2 \) and \( G = 1.55 \); Sc4, \( s_{r,H} = 0.2 \) and \( G = 1.55 \). Table 2 shows initial and final steady states.

Figure 1 Transitional Dynamics of \( D \), \( \bar{\eta} \) and \( W \)

Thus, in line with Fischer and Newell (2008) and Acemoglu et al. (2012), among others, we focus on two specific facts that are related with the post-1980 skill-biased wage inequality in developed and (newly-industrialised) developing countries: shift in the composition of public spending towards high-skilled Clean technological knowledge, through \( s_{x,H} \) and/or \( s_{r,H} \) and/or \( G \). Indeed, in line with Jaffe, Newell, and
Stavins (2005), R&D subsidies for Clean technology, $s_{x,H}$ and/or $s_{r,H}$, and EG&S provided by the government, $G$, should be used not only as substitutes, but also as complements. Moreover, R&D subsidies and EG&S with environmental objectives are quite common, including government-sponsored research programs, joint initiatives, grants, and tax incentives. Policies have indeed a significant impact on a country’s green patenting. Major programs exist in the United States, United Kingdom, Denmark, Ireland, Germany, Japan, and the Netherlands (e.g., Fischer and Newell 2008; Johnstone, Hascic, and Popp 2010). The aim is thus to analyse the effects of changes in $s_{x,H}$, $s_{r,H}$, and $G$ without being confused or distracted by other effects.

Environmental policies accentuate $D$: Sc1, Sc3 and Sc4 increase the size of profits to the producers of $j \in [1, 1]$, and Sc2 and Sc4 decrease the cost of $H$-specific R&D. Towards the new steady state, such bias increases the supply of $H$-intermediate goods, thus raising the use of the Clean sector, see (6), and lowering the relative $P$ price, see (7). $P$ drops continuously towards the steady-state, which implies that $D$ is rising, but at a falling rate. $D$ is thus motivated by the price channel, since there are stronger incentives to improve high-priced goods. The effect on $D$ is stronger through a direct R&D subsidy and without the level effect induced by $G$, due to the effect on $P$.

The competitiveness of the Clean sector is favoured in Sc2 and Sc4; in Sc2 mainly due to the path of $D$ and in Sc4 owing to the level effect; the same happens with $W$, since in Sc2 and Sc4 the relative demand for $H$ is strongly stimulated.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial steady-state values</th>
<th>Steady-state value under each Scenario, Sc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sc1</td>
<td>Sc2</td>
</tr>
<tr>
<td>$D$</td>
<td>1.56</td>
<td>2.54</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>0.49</td>
<td>0.43</td>
</tr>
<tr>
<td>$W$</td>
<td>1.55</td>
<td>1.98</td>
</tr>
</tbody>
</table>

Source: Own calculation, based on our computations.

5. Concluding Remarks

The literature related to environmental policy, technological knowledge and the resulting implications to economic growth reveals that, when technological knowledge is endogenously determined, environmental policy becomes very important to induce technological knowledge progress. However, there is no consensus about the endogenous growth effects of environmental policies. To better apprehend these effects, we develop a general equilibrium endogenous growth model in which final goods are produced either in the intensive Clean sector or in the intensive Unclean sector.

The former (latter) sector uses skilled (unskilled) intensive labour and a continuum set of specific quality-adjusted intermediate goods. In this scenario with complementarity between inputs and substitutability between sectors, numerical calculations describing dynamic equilibrium towards the steady state show that immediate, significant and stable environmental policies affect the technological-knowledge bias through the price-channel mechanism, which, in turn, affects positively the: (i) environmental quality; (ii) relative demand for skilled labour and thus the skill-premium;
and (iii) growth rate motivated by technological-knowledge progress. Moreover, Environment Goods and Services motivate an immediate level effect, whereas subsidies only affect the technological-knowledge bias.

Finally, we conclude that the effect of population growth on the steady-state economic growth rate relies positively on the households’ environmental care with future generations.
References


Appendix

In equilibrium, there will be a threshold final good \( \bar{n} \in [0,1] \), such that only \( L \) (\( H \)) labour will be used to produce final goods indexed by \( 0 \leq n \leq \bar{n} \) (\( \bar{n} < n \leq 1 \)). To prove this, let’s divide the profits of \( n, \Pi_n \), by \( L_n \) and by \( H_n \) to write the profit function per \( L \)- and \( H \)-labour, respectively:

\[
\Pi_n = p_n^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{q} \right)^{(1-\alpha)/\alpha} (1-n) Q_L \alpha - w_L
\]  

(A1)

\[
\frac{\Pi_n}{E_n} = p_n^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{q} \right)^{(1-\alpha)/\alpha} n G Q_H \alpha - w_H
\]  

(A2)

where \( w_m (m = L, H) \) is the price paid per labour unit. Since competition implies that in equilibrium \( \Pi_n = 0 \), every final good producer has to make zero profits per employee. Subtracting (A2) by (A1), we obtain an increasing function in \( n \):

\[
\frac{\Pi_n}{H_n} - \frac{\Pi_n}{L_n} = \alpha p_n^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{q} \right)^{(1-\alpha)/\alpha} [n G Q_H - (1-n) Q_L] - w_H + w_L.
\]

Thus, a threshold final good \( \bar{n} \in [0,1] \) must exist such that:

\[
\frac{\Pi_n}{H_n} - \frac{\Pi_n}{L_n} = 0 \quad \text{and} \quad \frac{\Pi_n}{H_n} - \frac{\Pi_n}{L_n} > 0 , \text{ for all } n > \bar{n} \), and
\[
\frac{\Pi_n}{H_n} - \frac{\Pi_n}{L_n} = 0 \quad \text{and} \quad \frac{\Pi_n}{H_n} - \frac{\Pi_n}{L_n} < 0 , \text{ for all } n < \bar{n} \),
\]

With \( \bar{n} \), the switch from \( L \)- to \( H \)-technology is advantageous and vice versa. Hence, taking into consideration both technologies separately, \( Y_n \) can be expressed by:

\[
Y_n = \begin{cases} 
A^{\frac{1}{\alpha}} p_n^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{q} \right)^{(1-\alpha)/\alpha} (1-n) Q_L = Y_n(L) , & 0 \leq n \leq \bar{n} \\
A^{\frac{1}{\alpha}} p_n^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{q} \right)^{(1-\alpha)/\alpha} n G Q_H = Y_n(H), & \bar{n} \leq n \leq 1
\end{cases}
\]  

(A3)

Also, in equilibrium, each labour type must be paid its marginal productivity that must be equalised across all \( n \in [0,\bar{n}] \) for \( L \)-labour and \( n \in [\bar{n},1] \) for \( H \)-labour; thus,

\[
w_L = p_n^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{q} \right)^{(1-\alpha)/\alpha} (1-n) Q_L
\]

\[
w_H = p_n^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{q} \right)^{(1-\alpha)/\alpha} n G Q_H.
\]
However, for marginal productivity to be equalized, $p_n^{\frac{1}{\lambda}}(1-n)$ and $p_n^{\frac{1}{\lambda}} n$ must be constants, i.e., they must be independent of $n$. Defining $p_L^{\frac{1}{\lambda}} = p_n^{\frac{1}{\lambda}} (1-n)$ and $p_H^{\frac{1}{\lambda}} = p_n^{\frac{1}{\lambda}} n$, the relative prices (price indexes ratio) of $M$ and $E$ final goods is:

$$P(t) \equiv \frac{p_H(t)}{p_L(t)} = \left[ \frac{\bar{\pi}(t)}{1-\bar{\pi}(t)} \right]^\alpha. \tag{A4}$$

Furthermore, in equilibrium, expenditures must also be equal for all final goods, i.e., $p_n Y_n$ must be constant for all $n$. Consequently, $p_n Y_n(L) = p_n Y_n(H)$. Using (A3) and (A4), the relative price of the two final goods satisfies:

$$P(t) \equiv \frac{p_H(t)}{p_L(t)} = \left[ \frac{\bar{\pi}(t)}{1-\bar{\pi}(t)} \right]^\alpha = \left( \frac{L_n Q_L}{G H_n Q_H} \right)^\alpha. \tag{A5}$$

Finally, combining (A4) with (A5) we obtain $\bar{\pi}$:

$$\bar{\pi}(t) = \left[ 1 + \left( \frac{Q_H(t)}{Q_L(t)} \frac{G H}{L} \right)^{\frac{1}{2}} \right]^{-1}. $$