A Flexible Nonlinear Inference to Okun’s Law for Turkish Economy in the Last Decade

Summary: The study applies the flexible nonlinear inference approach of James D. Hamilton (2001) to investigate the relationship between cyclical components of unemployment and output in the Turkish economy where the unemployment rate remains at double digits despite the relatively stable economic environment over the last decade. The paper shows that economic expansion and contraction terms have an asymmetric effect on cyclical unemployment in Turkey. Moreover, the study identifies a specific range for the output gap level at which jobless growth pattern occurs in the Turkish economy. According to our findings, contrary to standard literature, cyclical component of unemployment does not decrease even though cyclical component of output is positive and increases in the middle stages of the upswing phase of the economy. This result may also indicate that the employers are reluctant to extend employment and alter into informalization for reasons such as over-valued domestic currency, surplus labour force and/or any rigid regulatory frameworks in the middle stages of the expansion phase of the economy. However, they become eager to expand employment and renounce informalization only after a certain rate of economic growth is achieved.

Key words: Okun’s law, Flexible nonlinear inference, Jobless growth, Turkey.

JEL: C50, E24.

The capital account in Turkey was liberalized in 1989 and the Turkish economy suffered from several economic crises in 1991, 1994, 1999 and 2001. During this period, although various stabilization programs were implemented, high and volatile inflation accompanied by volatile economic growth persisted in Turkey. However, the Turkish economy has experienced a dramatic transformation after the Transition Program for Strengthening the Economy began to be implemented in May 2001. Despite the relatively stabilized economic environment over the last decade, some researchers have discussed Turkey’s slow job creation performance or the “jobless pattern of growth” (see, for example, Insan Tunali 2003; Mehmet T. Pamukcu and Erinç Yeldan 2005; Hakan Erkan and Aysıt Tansel 2006; Çağatay Telli, Ebru Voyvoda, and Yeldan 2006; Yeldan 2006, among others).
This paper aims to investigate the relationship between unemployment and GDP growth by using quarterly data for the period from 2001Q2 to 2013Q1 for the Turkish economy. We employed the methodology developed by Hamilton (2001) to the linear gap model to make flexible nonlinear inferences about Okun’s Law for the Turkish economy. Hamilton’s method provides both a test of the null hypothesis of linearity, which is based on the Lagrange multiplier principle, and a consistent estimation of what the nonlinear relation looks like. Our results indicate that unemployment and economic growth have a significant nonlinear relationship in Turkey. Economic expansion and contraction terms have an asymmetric effect on unemployment. Moreover, the study identifies a specific range for the output gap level at which a jobless growth pattern occur in the Turkish economy and contributes to the literature by empirically showing that the low job creation performance over the last ten years in Turkey coincides with medium economic growth rates.

The rest of this paper is organized as follows. Section 1 gives a brief literature review on asymmetries in Okun’s Law. Section 2 summarizes the past decade for the Turkish economy and outlines the literature on jobless growth for Turkey. Section 3 describes the methodological issues. Section 4 presents the empirical findings of the study. Section 5 contains our conclusion.

1. Asymmetries in Okun’s Law: A Brief Literature Review

One can measure the cost of higher unemployment by using Okun’s rule of thumb. Arthur M. Okun (1962) stated that a one per cent increase (or decrease) in the unemployment rate causes a three per cent decrease (or increase) in output. Of course, the empirical regularity is affected by a number of factors such as different labor market structures of different countries, different population growth rates and an increasing female labor force, tax policies, labor productivity, etc. However, a number of researchers provided support for the empirical validity of Okun’s study, finding a significantly negative coefficient of the relationship (for example, Robert J. Gordon 1984; Roger T. Kaufman 1988; Martin F. J. Prachowny 1993; Christian E. Weber 1995; Imad A. Moosa 1997). More recently, Laurence M. Ball, Prakash Loungani, and Daniel Leigh (2013) stated that Okun’s Law is a strong and stable relationship, one that did not substantially change during the Great Recession in most countries.

On the other hand, some authors focused on the possibility that asymmetries exist in this relationship. For instance, Jesus C. Cuaresma (2003) stated that the use of purely linear models could lead to misleading results. In further detail, Richard Harris and Brian Silverstone (2001) emphasize the importance of asymmetries in Okun’s Law for the following reasons. First, asymmetries in Okun’s Law play a critical role in alternative theories of joint labor and good market behavior. Second, asymmetry in Okun’s Law also indicates asymmetry in the Phillips curve. Third, knowledge of the present asymmetry is useful for both structural and stabilisation policies. Finally, if current asymmetries are ignored, forecasting errors arise.

Indeed, there are a number of theoretical explanations for asymmetries in the output-unemployment relationship. According to Hugh G. Courtney (1991), factor
substitutions during cycles, fluctuations in multi-factor productivity and changes in sectorial growth rates signify asymmetry. Similarly, Thomas I. Palley (1993) indicated that changes in sectoral growth rates and labor force participation rates cause asymmetry in Okun’s Law. According to him, from the labor demand side, the magnitude of Okun’s coefficient is linked to employment policies of firms, on the other hand form the labor supply side, it depends upon the labor force participation decisions of households. David Mayes and Matti Viren (2002) note the mismatch between jobs and the unemployed in terms of sectors and regions, especially during the downturn phase of the economy. Harris and Silverstone (2001) focus on asymmetric responses by heterogeneous plants in terms of job creation and job destruction performances due to external shocks. Paramsothy Silvapulle, Moosa, and Mervyn J. Silvapulle (2004) first assume that there are no restrictions on hiring or firing employees, and calls attention to the bad news - good news phenomena. When the economy enters into a downswing phase, firms respond to this shock by firing workers. However, employers are not willing to hire more workers in the upswing phase of the economy due to fear of the possibility that economic recovery might not continue for a long time. In other words, employers tend to be more pessimistic on the downturn phase than optimistic on the upturn phase which is consistent with risk aversion. They then allow for restrictions on firing or hiring (not too strong as firing) employees and investments in staff training. They argue that employers will be willing to hire more workers when the economy recovers. Therefore, a reverse scenario will be valid and the response of unemployment to output growth will be faster in the upswing than the downswing phase of the economy.

More recently, World Bank (2010) and Marek Hanusch (2012) pointed to another source of asymmetry, especially for developing economies. According to this point of view, Okun’s Law is reversed for agricultural jobs. In other words, a negative shock to growth results in more jobs in the agriculture sector and it may serve as a shock-absorber for employees who are laid off in industrial and service sectors when the economy slows down (Hanusch 2012). Hence, the overall effect of a negative shock to growth on unemployment is smoothed out by the agriculture sector.

Over the last decade, following the above mentioned arguments a number of empirical studies have taken the asymmetries in this relationship into account. Jim Lee (2000) evaluated robustness of Okun’s relationship for 16 OECD economies and reported that there is mixed evidence of asymmetric behavior. However, he showed that smaller output loss is associated with greater unemployment especially after the 1970’s in most countries. Harris and Silverstone (2001) demonstrated that the short-run output and unemployment adjustment to disequilibrium usually differs in terms of upswing and downswing phases of the economy in seven OECD countries. They also indicated that in most of the economies in their sample, the labor market continues to tighten in upturns when there is disequilibrium between unemployment and output. Cuaresma (2003) estimated a nonlinear specification of the relationship between cyclical unemployment and cyclical output for the US economy and he reported the regime-dependent Okun’s parameter which shows a significantly higher absolute value for recessions than for expansions. Ho-Chuan R. Huang and Shu-Chin Lin (2006) investigated the
nature of nonlinearities in Okun’s Law for US data by using the flexible approach to nonlinear inference developed by Hamilton (2001). They found significant evidence of nonlinearity. However, it is emphasized that, the relationship is still negative and provides strong support of (nonlinear) Okun’s Law. Mark J. Holmes and Silverstone (2006) using the Markov regime switching approach, found a significant inverse relationship between cyclical output and unemployment in expansionary regimes for the US data. Given their findings, they concluded that the notion of jobless recoveries in 1991 and in 2001 may be exaggerated. Mathieu Jardin and Stephan Gaétan (2012) employed a semi-parametric approach to describe Okun’s Law for 16 European countries. According to their findings, unemployment responds more strongly to output when the economy is contracting than expanding. Hence, they concluded that asymmetries in the relationship have to be taken into account when designing structural and stabilization policies. Policies in the downswing phase of the economy will be inadequate in the middle of recessions and during recoveries because of the regime dependent Okun’s coefficient.

2. Jobless Growth Path of the Turkish Economy in the Past Decade

During the 1990s, Turkish economy suffered from various problems such as the unsustainable public sector debt, structural problems in financial sectors and high inflation and low growth rates. The exchange rate-based stabilization program, which was adopted in December 1999 ended in the following crises of November 2000 and February 2001. Following these crises, a new stabilization program was implemented under a flexible exchange rate regime. The main goal of this stabilization program, which was also signed with the International Monetary Fund (IMF) in May 2001, was to decrease the inflation and the public deficit. As a result, the program focused not only on concretionary fiscal policy but also on improving the financial and banking sectors. In early 2002, Central Bank of Turkey (CBT) began to implement an inflation targeting regime, and inflation remained at single digit rates since 2004. The fiscal dominance has been significantly reduced, for instance, the public debt to GDP ratio, which was 91.1% at the beginning of 2002 decreased to 59.9% at the end of 2006 (see Hasan Ersel and Fatih Özatay 2008). The IMF agreement was renewed in May 2005, and resuming the fiscal and the monetary discipline was presented as the main aim of this program as well. CBT began to implement an explicit inflation targeting regime at the beginning of 2006. The integration of the Turkish economy into international markets has been increasingly accelerated by means of factors such as price stability, increase in credibility, decrease in risk premiums and political stability (see, Rauf Gönenç, Saygin Sahinöz, and Ozge Tuncel 2010). Moreover, thanks to the sound financial system and the absence of fiscal dominance and structural reforms, the Turkish economy grew by approximately 7% between 2002 and 2007. This high growth path of the Turkish economy had prolonged until the eruption of the global financial crisis in 2009. Nonetheless, economic growth was at 9.16% in 2010 and 8.77%, which was the second highest economic growth rate worldwide in 2011.
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On the other hand, over the same period, the unemployment level in Turkey remained above the 10% level as seen in Figure 1. The global financial crisis in the last term of 2008 had a negative impact on unemployment, increasing to 11% and dramatically rising to 14% just one year later. As of 2010, the unemployment rate decreased to 11.9% and to 9.8% in 2011. On the contrary, growth rate of the economy declined sharply to 2.2% in 2012, and unemployment rate increased above 10%. It is clear that there is an asymmetric relationship between output growth and unemployment rate and the effect of output growth on unemployment is much stronger in the downswing phase than the upswing phase of the economy.

On the other hand, a group of researchers submit that employment and growth do not have to move together (see for example, Mario Pianta and Rinaldo Evangelista 1996; Paolo Piacentini and Paolo Pini 2000, among others). In compliance, according to Figure 1, one may also claim that it seems as if the unemployment rate is insensitive to GDP growth, especially in the upswing phase of the economy. In fact, Recep Tari and Tezcan Abasiz (2010) investigated the asymmetry in Okun’s Law for Turkey between 1968 and 2008 by using two-regime threshold vector error correction model and indicated that the impact of growth on unemployment is asymmetrical. They also reported that the unemployment rate was not affected by the fluctuations in growth in the upswing phase of the economy and this situation indicates the jobless recovery. Similar results also have been reported for the Turkish economy by several different authors. For example, Hüseyin M. Yüceol (2006) stated that there is no long-run causality between economic growth and unemployment between 1950 and 2004 in Turkey. Oner Guncavdi and Suat Kucukciftci (2004) stated that parallel to the decrease in labor in time, there are important decreases observed within the employment creation power of the production increases between 1973 and 1996. These authors also emphasize that it is not that difficult to estimate unemployment in the 2000s. Senay Açıkgoz and Merter Mert (2010) empirically show that the natural rate of growth in Turkey is

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endogenous hence economic growth can be stopped due to demand constraints before reaching the full employment ceiling.

The related literature examines this issue through three perspectives for the Turkish economy, and the informality of formal employment is emphasized. The first perspective suggests that the open capital account and the international financial speculation beginning in the 1990s and continuing over the last decade have caused the traditional labor intensive import industry to undergo a serious change (see, for example, Telli, Voyvoda, and Yeldan 2006; Yeldan 2006, among others). According to the authors in favor of this point of view, the main source of this change is the sizeable appreciation of the local currency due to the short-term financial capital attracted by a high interest rate and by increased importation (particularly, the importation of intermediate goods), significantly reducing the traditional labor-intensive exports. Hence, it causes the contractions of formal jobs and leads to informalization. Second, high labor costs due to high taxes on wages, high non-wage labour costs, low productivity and rigid regulatory frameworks became salient (see, for example, Tunali 2003). According to this viewpoint, high taxes and employment protection laws, etc. may not only cause employers to be reluctant about expanding employment but it may also encourage informalization. On the other hand, there are empirically supported opposing views related to the high degree of labor market rigidity for Turkish economy (see, for example, OECD 2000; Robert A. Lawson and Edward Bierhanzl 2004). Additionally, Özlem Onaran (2002) indicated that wages have a high degree of flexibility because the power of trade unions has been corroded since the liberalization policies have begun to be implemented, thus, she argued that unemployment is not assumed to be a labor market rigidity problem for the Turkish economy. Moreover, Erol Taymaz and Şule Özler (2003) rightly argued that the degree of labor market flexibility increases when the informalization (informal sector and informalized practices in formal sector) is considered. Third, it is a well-known fact that agriculture’s share in output and employment declines because of economic development (see, for example, Arthur W. Lewis 1954; Simon Kuznets 1966). This movement of the labor force from agriculture sector to industrial and service sectors is another source of change. For instance, Ercan and Tansel (2006) indicate that the movement of workers from the agriculture sector creates the informalization of urban markets in Turkey. Similarly, İpek Ilkaracan and Tunali (2009) stated that while agriculture’s share in total employment was equal to 36% in 2000, it decreased to 25% in 2008 and during that time two million inhabitants were added to the urban surplus labor category in the Turkish economy. On the other hand, some sociologically based explanations for this issue can be found within the scope of the “rural-urban continuity” literature for Turkey. Briefly, this literature emphasizes the continuity rather than spatial fracture within the rural workforce changing into an urban form for Turkey. Suzan M. Ilcan (1994), for instance, gives examples that the rural-urban migration did not disconnect the immigrant from the rural, from the point of the immigrant. This connection is in the ascendant, particularly during the slowdowns. Again, the cohesive power of the family can create alternative employment forms in both urban and rural areas (Kezban Celik 2008). In this sense, it seems possible to observe the shock-absorber characteristic of the agricultural sector of the Turkish
economy. Likewise, Tansel (2012) reported that the decrease in agricultural employment was reversed between 2008 and 2011 experiencing a significant increase for three years in Turkey.

All in all, the relationship between unemployment and growth is a puzzle for the Turkish economy. Arthur C. Pigou (1993) stated that unemployment is not the sum of separate factors, but it is instead the simultaneous result of a system of interconnected factors. Hence, in our viewpoint, none of the previously mentioned factors alone explains the issue at hand, but somehow they are all jointly responsible for it in the various phases of the business cycle in the Turkish economy.

3. The Methodological Issues

The two equations that Okun used in his 1962 paper “Potential GNP: Its Measurement and Significance” have been popular in the literature. First one is the difference model in which the relationship between observed unemployment rate and natural log of observed real GDP is expressed as their first differences. The second equation is the gap model, where the relationship between cyclical component of unemployment \( (ur^c) \), or unemployment gap, and cyclical component of log real GDP \( (y^c) \), or output gap, is investigated. Of these two Okun’s Law specifications, we have opted for the “gap” model.

3.1 Theoretical Background

Let us specify an aggregate production function as follows:

\[
Y = AF(K, L),
\]

where, \( Y \) is the output level, \( A \) is the measure of ability of transforming inputs into outputs, \( K \) is the capital and \( L \) is the labor input. In further detail, Equation (1) can be specified as:

\[
Y = AF(kxc, nxh),
\]

where \( k \) is the number of capital stock, \( c \) is the utilization rate, \( n \) is the number of workers and \( h \) is the number of working hours. If one differentiates Equation (2) with respect to \( t \) (time), yields:

\[
\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \left( \frac{\partial F}{\partial K} \right) \left( \frac{k}{k} + \frac{\dot{c}}{c} \right) + \left( \frac{\partial F}{\partial L} \right) \left( \frac{n}{n} + \frac{\dot{h}}{h} \right).
\]

Let say \( \dot{u} \) is the change in the unemployment rate which is determined by using the following approximation:

\[
\dot{u} \approx \frac{\dot{l}}{l} - \frac{n}{n}, \tag{4}
\]

where, \( l \) denotes the labor stock. Hence, Equation (3) can be expressed as:

\[
\dot{y} = \frac{\dot{A}}{A} + \alpha^* \left( \frac{k}{k} + \frac{\dot{c}}{c} \right) + \beta^* \left( \frac{l}{l} - \dot{u} + \frac{\dot{h}}{h} \right), \tag{5}
\]
where, \( \dot{y} = \frac{\dot{Y}}{Y} \), \( \alpha^* = \left( \frac{\partial F}{\partial K} \right) \) denotes the elasticity of output with respect to capital and \( \beta^* = \left( \frac{\partial F}{\partial L} \right) \) denotes the elasticity of output with respect to labor, respectively. Solving Equation (5) for \( u \) yields:

\[
\dot{u} = -\frac{1}{\beta^*} \dot{y} + \frac{1}{\beta^*} \left( \frac{\dot{A}}{A} + \alpha^* \left( \frac{k}{k} + \frac{c}{c} \right) \right) + \left( \frac{l}{l} + \frac{\dot{u}}{\dot{u}} \right),
\]

Equation (6) here, if the rate of technological progress, capital growth, capital utilization, labor force, and average working hours vary with output growth and their relationships with \( \dot{y} \) change over time as, \( \frac{\dot{A}}{A} = f_1(\dot{y}), \frac{k}{k} = f_2(\dot{y}), \frac{c}{c} = f_3(\dot{y}), \frac{l}{l} = f_4(\dot{y}), \frac{h}{h} = f_5(\dot{y}) \), where, \( f_j(\dot{y}), j = 1,2,\ldots,5 \) denotes the unknown relationship between these variables and \( \dot{y} \). Inserting these equations into Equation (6) yields Okun’s relationship in the following form:

\[
\dot{u} = \beta \dot{y} + f(\dot{y}).
\]

In Equation (7), there are two channels of output growth on \( \dot{u} \). First one is the direct effect of \( \dot{y} \) measured by \( \beta \) which is equal to \(-\frac{1}{\beta^*}\). The second one is the indirect effect of \( \dot{y} \) through its impact on technological process, capital growth, capital utilization, labor force, and average working hours, measured by the function \( f(\cdot) \) which can be treated as a composite function of \( f_j(\dot{y}) \). Of course, the functional form of \( f(\cdot) \) is unknown. However, it can be estimated by Hamilton’s (2001) flexible nonlinear inference methodology as presented in Subsection 3.3 (see, also Huang and Lin 2006).

### 3.2 The Gap Model

To remove the serial correlation and allow for short-run dynamics, Weber (1995), Moosa (1997), Cuaresma (2003) and Huang and Lin (2006) considered following an autoregressive distributed lag model as a linear gap model:

\[
ur_t^c = \alpha_0 + \sum_{i=1}^{p} \alpha_i ur_{t-i}^c + \beta y_t^c + \varepsilon_t,
\]

where \( \varepsilon_t \) is an error term at time \( t \), \( y_t^c \equiv y_t - y_t^* \), \( ur_t^c \equiv ur_t - ur_t^* \), \( y_t^* \) represents the potential level of log real GDP and \( ur_t^* \) is a natural rate of unemployment. Optimal lag length (\( p \)) for the model can be determined by using Akaike’s Information Criterion or Bayesian Information Criterion.

In this specification the major problem is the estimation of \( y_t^* \) and \( ur_t^* \) since there are no observable data for these variables. To overcome this problem, by following Cuaresma (2003) and Huang and Lin (2006), we used Andrew C. Harvey’s (1989) structural time series model, where the filtering approach applied to the data is used to decompose the observed series into an unobserved trend and cyclical components. In particular, the basic model can be formulated as:
where \( z_t \) is the observed series, \( z_t^T \) and \( z_t^C \) are the unobserved trend and cyclical components and \( \epsilon_t \) is an irregular component that is assumed to be white noise. Since we used seasonally adjusted data, the seasonal component is excluded from the model.

The long-term movement of a series, representing the potential value of the underlying variable, is that the trend component, \( z_t^T \), which is assumed to be stochastic and linear, can be represented as:

\[
\begin{bmatrix}
  z_t^T \\
  \kappa_t
\end{bmatrix} = \begin{bmatrix}
  1 & 1 \\
  0 & 1
\end{bmatrix} \begin{bmatrix}
  z_{t-1}^T \\
  \kappa_{t-1}
\end{bmatrix} + \begin{bmatrix}
  \eta_t \\
  \zeta_t
\end{bmatrix},
\]

where \( \eta_t \sim \text{NID}(0, \sigma_\eta) \), \( \zeta_t \sim \text{NID}(0, \sigma_\zeta) \), \( z_t^T \) is a random walk with drift factor, \( \kappa_t \) which follows a first order autoregressive process. The cyclical component, \( z_t^C \) is assumed to be a stationary linear process and can be represented as:

\[
\begin{bmatrix}
  z_t^C \\
  z_t^{C*}
\end{bmatrix} = \rho \begin{bmatrix}
  \cos \lambda & \sin \lambda \\
  -\sin \lambda & \cos \lambda
\end{bmatrix} \begin{bmatrix}
  z_{t-1}^C \\
  z_{t-1}^{C*}
\end{bmatrix} \begin{bmatrix}
  w_t \\
  w_t^{*}
\end{bmatrix}.
\]

After writing the model in state space form, the likelihood function can be calculated by using the Kalman filter approach and hyper parameters \( (\sigma_\eta, \sigma_\zeta, \sigma_{\omega}^2, \lambda, \rho) \) estimated by the maximum likelihood approach. As a result, one can reach the estimates of \( z_t^T \) and \( z_t^C \), respectively.

### 3.3 Flexible Nonlinear Inference

The flexible modelling approach allows us to determine whether a relation is nonlinear. In particular, Hamilton (2001) suggests a novel model of the form:

\[
y_t = \mu(x_t) + \epsilon_t, \quad t = 1, 2, \ldots, T,
\]

where \( y_t \) is the dependent variable, \( x_t \) is a vector of \( k \times 1 \) explanatory variables and \( \epsilon_t \) is the error term independently distributed \( N \sim (0, \sigma^2) \). Following William E. Wecker and Craig F. Ansley (1983), the basic idea underlying Hamilton’s methodology is not only to treat the dependent variable, \( y_t \), as a realization of a stochastic process, but also to consider the functional form of the conditional mean function \( \mu(x_t) \) as the outcome of a random process, which is assumed to be a combination of a linear part and a stochastic nonlinear part:

\[
\mu(x_t) = \alpha_0 + \alpha'(x_t) + \lambda m(g \otimes x_t),
\]

where the symbol \( \otimes \) denotes the element-wise product of the matrices and \( m(\cdot) \) is the latent outcome of the random field. The term \( \lambda \) contributes to nonlinearity whereas \( g \) controls the curvature.

Moreover, for any choice of \( x \), \( m(x) \) is a realization from random field with distribution as:
\[ m(x) \sim N(0,1), E[m(x)m(z)] = H_k(h), \]

where \( h = \frac{1}{2}[(x - z)'(x - z)]^{\frac{1}{2}} \) is based on the Euclidian distance and \( H_k(\cdot) \) is specified as:

\[
H_k(h) = \begin{cases} 
G_{k-1}(h, 1) / G_{k-1}(0,1) & \text{if } h \leq 1 \\
0 & \text{if } h > 1 
\end{cases}
\]

with \( r \geq h \geq 0 \) and \( G_k(h, r) = \int_h^r (r^2 - z^2)^{k/2} \, dz \).

Then, Hamilton (2001) shows that \( G_k(h, r) \) can be obtained recursively and provides closed form expressions for \( H_k(h) \) for \( k = 1, \ldots, 5 \).

In the estimation procedure, because \( m(\cdot) \) is latent, the conditional mean function \( \mu(x_t) \) and the parameter vector \( \theta = (\alpha_0, \alpha', \lambda, g, \sigma) \) do not provide any inference. Hamilton (2001) shows how to obtain the ML estimate of the parameters \( \theta \) using a recursive algorithm similar to that of the Kalman filter used to obtain the ML estimates of the state space model. However, he also introduces an equivalent approach to obtain the GLS estimates. In particular, Hamilton accomplishes this approach by rephrasing Equations (13) and (14) as:

\[
y = X\beta + u, \tag{16}
\]

where,

\[
y = \begin{bmatrix} y_1 \\ \vdots \\ y_T \end{bmatrix}; X = \begin{bmatrix} 1 & x_1' \\ \vdots & \vdots \\ 1 & x_T' \end{bmatrix}; \beta = \begin{bmatrix} \alpha_0 & \alpha' \end{bmatrix}; u = \begin{bmatrix} \lambda m(g ? x_1) + \varepsilon_1 \\ \vdots \\ \lambda m(g ? x_T) + \varepsilon_T \end{bmatrix}
\]

they are conditional upon an initial set of parameters \( \lambda \) and \( g \). By defining \( \zeta = \frac{\lambda}{\sigma} \) and \( W(X; g, \zeta) = \zeta^2 H + I_T \), the parameters of the linear part, i.e., \( \beta \) and \( \sigma^2 \) can be calculated analytically as:

\[
\bar{\beta}(g, \zeta) = [X' W(X; g, \zeta)^{-1} X]^{-1} [X' W(X; g, \zeta)^{-1} y], \tag{17}
\]

\[
\sigma^2(g, \zeta) = \frac{\{y - x\bar{\beta}(g, \zeta)\}' W(X; g, \zeta)^{-1} [y - x\bar{\beta}(g, \zeta)]}{T}, \tag{18}
\]

where \( H(g) \) denotes \( T \times T \) matrix with a \((t, s)\) element of \( H_k(h_t(g)) \) and \( I_T \) is a \( T \times T \) identity matrix. Therefore, one can obtain the estimate of parameter vector \( \theta \) by maximizing the following concentrated log likelihood function in Equation (19) with respect to \( (g, \zeta) \) and the final estimates of \( (\beta, \sigma^2) \) can be found by plugging the values of \((\bar{g}, \bar{\zeta})\) into Equations (17) and (18):

\[
\eta(g, \zeta; X, y) = -\frac{T}{2} \ln(2\pi) - \frac{T}{2} \ln \sigma^2(g, \zeta) - \frac{1}{2} \ln |W(X; g, \zeta)| - \frac{T}{2}. \tag{19}
\]

Under such circumstances, Hamilton (2001) proposed the LM statistic below:

\[
\nu^2 = \frac{[\bar{\xi}' \bar{H} - \bar{\sigma}^2 tr(MHM)]^2}{\bar{\sigma}^4 (2tr[MHM - tr(MHM/\bar{M}M/HM/\bar{T} - k - 1))^2]}, \tag{20}
\]

where \( \bar{\xi} \) is a \( T \times 1 \) residual vector, which can be obtained from performing a linear regression of \( y_t \) on \( (1, x_t')' \), and \( \bar{\sigma}^2 \) is an estimated OLS variance. The matrix \( M = \)
\(I_T - X(X'X)^{-1}X'\) for \(X\) the \(T \times (k + 1)\) matrix whose \(i^{th}\) row is \((1, x_i')\) and the \((i,j)\) element of the matrix \(H\) is given in Equation (15). Under null which means that the relation is linear, the LM statistic \(v^2\) is \(\chi^2\) distributed with one degree of freedom.

4. Empirical Results

The data set used in this study was downloaded from the website of the Central Bank of the Republic of Turkey (2013). Official quarterly unemployment data for the Turkish economy has been published 2000, and we have opted to perform the analysis for the period beginning with the Transition Program for Strengthening the Economy. Before making nonlinear inferences about Okun’s Law, we first estimated the linear gap model. As mentioned in Subsection 3.1, we used Harvey’s (1989) structural time series model to decompose the observed series into an unobserved trend and cyclical components. Detrended components of series are shown in Figure 2. Roughly speaking, it seems as cyclical components of unemployment and (log) real GDP move in opposite directions.

Below we presented the estimation of the linear model, Equation (8), by the OLS approach with Newey West heteroscedasticity autocorrelation consistent standard errors in parentheses. Except for the constant term, all coefficients are statistically significant at 5% significance level. The negative coefficient of \(y_t^c\) indicates the validity of Okun’s Law. In addition, Table 1 indicates that the model passes diagnostic tests.

\[
u_t r^c = 0.002 + 0.54 u_{t-1}^c - 0.50 u_{t-2}^c - 0.26 y_t^c + \varepsilon_t
\]

\((0.005)\) \((0.18)\) \((0.15)\) \((0.12)\)

Source: Authors’ own calculations based on data from the Central Bank of Republic of Turkey (2013).

Figure 2 Time Series Plots of Cyclical Output and Cyclical Unemployment

Table 1  Diagnostic Tests for Linear Model

<table>
<thead>
<tr>
<th>Test</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.62</td>
</tr>
<tr>
<td>LM test (serial correlation)</td>
<td>$F(2,40) = 0.08$ [0.92]</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>$\chi^2(2) = 0.27$ [0.87]</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td></td>
</tr>
<tr>
<td>Breusch-Pagan-Godfrey</td>
<td>$F(3,42) = 1.44$ [0.25]</td>
</tr>
<tr>
<td>White</td>
<td>$F(9,36) = 1.90$ [0.09]</td>
</tr>
<tr>
<td>ARCH</td>
<td>$F(2.41) = 2.02$ [0.09]</td>
</tr>
<tr>
<td>AIC</td>
<td>-4.01</td>
</tr>
<tr>
<td>SIC</td>
<td>-3.83</td>
</tr>
</tbody>
</table>

Notes: Numbers in [ ] parenthesis indicate $p$-values.

Source: Authors’ own calculations.

Alternatively, Hamilton’s (2001) flexible nonlinear inference approach is used both to test for any nonlinearity in the given relationship and, if so, to see the form of the nonlinearity. Accordingly, we first employed the nonlinearity test by using the LM statistic, Equation (20), which yields a $\chi^2$ value of 5.67 with a corresponding $p$-value of 0.017. Hence, it is clear that the null hypothesis of linearity for $\mu(\cdot)$ can be rejected.

The maximum likelihood estimates of Equations (13) and (14) are represented as follows:

$$ur_t^c = 0.003 + 0.55ur_{t-1}^c - 0.51ur_{t-2}^c - 0.28y_t^c$$

(0.04) (0.17) (0.16) (0.13)

$$+1.50 \times 10^{-6}[2488.91m(56.86ur_{t-1}^c, 45.76ur_{t-2}^c, 94.83y_t^c) + v_t]$$

(4.5 \times 10^{-7}) (1034.65) (13.67) (12.22) (28.98)

where, $v_t \sim NID(0,1)$, $\varepsilon_t$ in Equation (13) is equal to $\sigma = 1.50 \times 10^{-6} \times \nu_t$ and $\lambda$ in Equation (14) is equal to $\sigma$ times the estimate 2488.91. In the linear part, it can be seen that $ur_{t-1}^c$, $ur_{t-2}^c$ and $y_t^c$ are statistically different from zero at 1% significance level and that they are very similar to the estimates of the linear gap model. In the nonlinear part, the estimates of $ur_{t-1}^c$, $ur_{t-2}^c$ and $y_t^c$ are 56.86, 45.76 and 94.83 with corresponding $t$ values 4.16, 3.74 and 3.27, respectively. These results imply that there is a statistically significant nonlinear part.

Given the empirical evidence of nonlinearity, to see what the nonlinear function $\mu(\cdot)$ looks like, we plotted the conditional expectation function with respect to $y_t^c$, i.e. Okun’s coefficient, while holding $ur_{t-1}^c$ and $ur_{t-2}^c$ fixed by their sample means. The line in Figure 3 displays the estimated function of Equation (14) evaluated at $x_t = (y_t^c, ur_{t-1}^c, ur_{t-2}^c)$ which represents the effect of $y_t^c$ (horizontal axis) on $ur_t^c$ (vertical axis). Figure 3 can be analyzed in three segments. The first is the terms that $y_t^c$ takes that are lower than 1%. The second segment involves the terms that $y_t^c$ takes that are between 1% and 2.2%. The third segment points to terms that $y_t^c$ takes that are higher than 2.2%.

Among these segments, a higher $y_t^c$ value in the first and third segments corresponds to a lower $ur_t^c$ and vice versa and indicates the validity of Okun’s Law. Additionally, the slope of the estimated function is less in segment 3 than in segment...
1. Hence, this situation illustrates the asymmetry indicated in the related literature as mentioned in the literature review section of this paper.

![Conditional Expectation Function with Respect to $y_t^c$](image)

\textbf{Figure 3} Conditional Expectation Function with Respect to $y_t^c$

Examination of the second segment in Figure 3 presents some interesting results. $ur_t^c$ takes on negative values and is consistent with $y_t^c$ which takes on positive values within the whole second segment. However, it is interesting that the cyclical component of unemployment is negative but does not decrease even though the cyclical component of output is positive and increases and even when $y_t^c$ reaches the 2.2% level, $ur_t^c$ is nearly equal to zero. Moreover, in order to recover the decreases in the unemployment gap, the output gap should reach level of approximately 4.5% (see dotted horizontal line in segment 3 in Figure 3). Hence we claim that the study identifies a specific range for the output gap level at which a jobless growth pattern occurs in the Turkish economy. According to our results, the jobless growth performance over the last ten years in Turkey coincides with the middle stages of the upswing phases of the economy. This result may also indicate that although in the middle stages of the economic expansion, employers are reluctant to extend employment and move towards informalization for reasons such as over-valued domestic currency, surplus labour force and/or any rigid regulatory frameworks in the Turkish labour market, they become eager to expand employment and renounce the informalization when a certain rate of economic growth is achieved.

Of course, this explanation only covers the increases in $y_t^c$ in the second segment. On the other hand, when $y_t^c$ decreases, $ur_t^c$ also decreases in the second segment. A possible explanation for this unexpected movement of unemployment may be related to the agriculture sector’s shock-absorber characteristic. In other words, in the second segment, when $y_t^c$ increases, it creates informal jobs in industrial and service sectors, on the other hand, agriculture sector may serve as a shock-absorber for employees when the economy slows down.
5. Conclusion

This study analyzes the relationship between the unemployment gap and the output gap in Turkey by applying Hamilton’s (2001) flexible nonlinear inference methodology to Okun’s widely popular gap model. Using the quarterly data set for the period in which the Turkish economy was in a state of flux, this study evaluates the stable economic environment that Turkey experienced despite the low job creation performance.

On the basis of our findings, we conclude that the relationship between the unemployment gap and the output gap is significantly nonlinear. Additionally, we defined that the effect of the output gap on the unemployment gap can be considered in three segments. Among all these three terms, a higher $y_1^c$ corresponds to a lower $u_r^c$ within the first and the third segments, and the job creation potential in the third segment is lower than the unemployment created by the recession in the first segment. Moreover, during the second segment, the cyclical component of unemployment does not decrease even though the cyclical component of output is positive and increases; and even when the output gap reaches the 2.2% level, the unemployment gap is nearly equal to zero. In addition to this, to recover the decreases in the unemployment gap, the output gap should reach the level of approximately 4.5%. Hence, we claim that the employers, who are reluctant to extend employment and alter into informalization, are eager to expand employment and renounce the informalization only after a certain rate of economic growth is achieved. This result is also consistent with the risk aversion of employers which is emphasized by Silvapulle, Moosa, and Silvapulle (2004). Moreover, in Turkey’s case, the result of risk aversion is not only observed as an asymmetrical response of the unemployment gap to the output gap in upswing and downswing phases; another kind occurs unexpectedly during the middle stage of the upswing phase of the economy due to the direction of monetary policy and the labor market condition. During the middle stages of upswing phases, employers are not optimistic or convinced enough to expand employment. As suggested by the related literature, overvalued domestic currency, surplus labor force with corroded power of trade unions and/or rigid regulatory frameworks in the labor market are opening a door to informal sector and informalized practices in formal sectors. In addition, the agriculture sector may have a shock absorber characteristic during slowdowns in the Turkish economy. All in all, one may claim that jobless growth performance over the last ten years in Turkey coincides with the middle stages of the upswing phases of the economy. However, the relationship between output and unemployment is still negative which supports of (nonlinear) Okun’s Law.

Consequently, the findings received by applying Hamilton’s (2001) flexible nonlinear inference methodology provide a useful framework, not only for policy makers in implementing new structural and stabilization policies, but also for researchers engaged in studies on the Turkish monetary policy. In light of our findings, it is clear that asymmetries have to be considered in terms of regime dependent Okun’s coefficient. Otherwise, it is inevitable that there will be policy discrepancies over the business cycle. As mentioned in Section 1, there is a theoretical linkage between Okun’s Law and the inflation-output trade-off. In this sense, our results imply a shift in the sacrifice ratio which is independent from sign of the output gap and points out a
three-regime model. Obviously, our sample size is a serious constraint for that kind of attempt in this study. Finally, another potential limitation of this study is that the issue at hand was analyzed on the basis of Okun’s Law perspective. The alteration in the Turkish labor market should be addressed in also employment front. This is especially important for researchers engaged in labour market studies. Nonetheless, we can claim that further studies on above mentioned issues should focus on the middle stage of the upswing phase where the unexpected relationship between unemployment and economic growth occurs.
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