

Guglielmo D'Amico

Department of Drug Sciences,
University "G. D'Annunzio", Chieti,
Italy

✉ g.damico@unich.it

Giuseppe Di Biase

Department of Drug Sciences,
University "G. D'Annunzio", Chieti,
Italy

✉ dibiase@unich.it

Raimondo Manca

Department of Mathematics for the
Economics, Financial and Insurance
Decisions,
University "La Sapienza", Rome,
Italy

✉ raimondo.manca@uniroma1.it

Work supported by MIUR research grant PRIN 2008, "Risparmio pensionistico e benefici previdenziali privati: scelte individuali, rischi per il gestore" ("Retirement savings and private pension benefits: individual choices, risks for the operator"). Scientific director: prof Ermanno Pitacco, University of Trieste, Italy.

Effects on Taxation on the Forecasting of Income Inequality: Evidence from Germany, Greece, and Italy

Summary: In this paper, we investigate the impact of the fiscal system on wealth redistribution in Germany, Greece, and Italy. We demonstrate the application of the model to the data of the quoted countries. We obtain the gross income distributions by starting from the net income distributions downloaded from the Eurostat website and by using the individual income tax rates of each country. We evaluate the Dynamic Theil's Entropy that allows us to recover the total inequality between the net and gross income distributions for each of these countries. Such a comparison allowed us to understand how the fiscal systems affect wealth distribution. These results can be used for planning welfare policies.

Key words: Income distribution, Dynamic Theil's entropy, Fiscal system, Welfare policies.

JEL: E64, E27.

Income inequality can be measured by means of econometric indices. The most common are the Gini index and Theil's entropy.

Henri Theil (1967) introduced Theil's Entropy as an alternative measure of income inequality. This index is popular because of its several desirable properties. Of these properties, we use additively decomposability inequality to obtain the values of the indices related to the three European countries of interest. In the whole population, the Theil's index can be additively decomposed to the inequality between the subgroups and to a weighted average of the inequality within each group.

Guglielmo D'Amico and Giuseppe Di Biase (2010) recently generalized Theil's Entropy by taking into account the random time evolution of the agents' incomes. This led to the formulas for the general dynamic inequality and the concentration indices.

In this paper, we investigate the impact of the fiscal system on wealth redistribution in the population for each of the considered countries. We could recover the gross income distributions by using the individual income tax rates in these countries over the net income distributions. Then, by following the methodological approach by D'Amico, Di Biase, and Raimondo Manca (2012), we compute the Dynamic Theil's Entropy.

Fiscal policy could be a fundamental tool of macroeconomic policy that can decrease income inequality by favoring the redistribution of the wealth.

The paper is organized as follows. Section 2 briefly describes the formula for Dynamic Theil's Entropy. Section 3 presents the data and the methodology of the application of the model. Section 4 shows the results. Conclusions are provided in Section 5.

1. Literature Survey

In literature, we find two lines of research on the study of the changes of income distribution. One line of research regards the study of static econometric inequality indices, whereas the other proposes income dynamic models.

The themes of inequality and wealth concentration in a subclass of an economic system have acquired high relevance and they have been studied extensively.

Anthony B. Atkinson (1970) specified the form of the social welfare function to emphasize that any measure of inequality involves judgments about social welfare.

François J. Bourguignon (1979) deeply studied the Theil's coefficient and the logarithm of the arithmetic mean over the geometric mean. Frank Cowell (1980) investigated many general decomposable measures.

Anthony F. Shorrocks (1984) proved that the decomposable inequality measures must be monotonic transformations of the additively decomposable indices under weak assumptions.

More recently, Oscar Bajo and Rafael Salas (2002) provided a connection between concentration and inequality measures by putting a particular emphasis on the general entropy inequality indices. These theoretical aspects were developed by Casilda L. De La Vega and Ana M. Urrutia (2005) and Hamid Shahrestani and Bijan Bidabad (2010). The former generalized the decomposability notion and studied those inequality measures that admit a path independent multiplicative decomposition. The latter designed a model to estimate the functional parameters of the Lorenz curve. The Lorenz curve is a graphical representation of the cumulative income distribution function.

Among the significant applications we mention the study by George Athanopoulos and Farshid Vahid (2003) on the changes in income inequality of individuals in Australia by assessing their statistical significance through the bootstrap method. Moreover, Xander Kooleman and Eddy Van Doorslaer (2004) added an intuitive understanding of the concept of the index for measuring relative inequality with an application of health-related measures by income and, finally, Ranjpour Reza and Karini T. Zahra (2008) analyzed income convergence in ten of the new members of the European Union toward the average European Union per capita income.

In parallel, the other research theme concerns modelling the dynamics of income time. The most common approach relies on the use of the Markov chain.

In particular, in a series of papers, Danny T. Quah (1993, 1994, and 1996) proposed the use of the Markov chain to verify the wealth convergence hypothesis and to estimate the transition matrix for the world's cross-country distribution of per capita incomes.

Frank Bickenbach and Eckhardt Bode (2003) tested the Markov chain hypothesis of income dynamics by using data for the states of the US from 1929 to 2000.

With the aim of producing an approach that unifies the previous two lines of research, D'Amico and Di Biase (2010) proposed the use of a semi-Markov process to compute inequality indices dynamically. Indeed, with the dynamic inequality indices, it is possible to justify the changes in the indices even when the population composition varies over time, which is a limitation of the static inequality measures. Actually, the static measures represent a simple snapshot of the economy and further, they are unable to capture the source of income variability over time. Additionally the static approach disregards the dependence within the sequences of incomes and is not able to describe the population dynamic given that it does not make use of a transition probability matrix that describes the agents' income evolution.

The economic inequality measures provide information that can be useful to promote the process of economic integration among different countries. A concrete example is the process of European integration, which was well exposed by Božo Stojanović (2011).

Moreover, it is well recognized that economic integration can be promoted by an adequate fiscal policy, see for example, Miroslav Prokopijević (2010) and Philip Arestis (2011).

Based on the above subjects and the related literature, in this paper we try to fill the gap by deriving an income dynamic model that takes into account the effects of fiscal policy on wealth inequality.

2. Dynamic Theil's Entropy

Nowadays, it is important to forecast income inequality. A simple way to achieve this is by considering the pattern of a generic static inequality index, such as, Theil's Entropy.

However, the use of the dynamic version of the Theil's index is highly instructive with respect to the consideration of the pattern of the static inequality index. The static approach indeed presents at least two disadvantages with respect to our approach.

First, it disregards the dependence within the sequences of incomes owing to the presence of an underlying Markov chain that drives income evolution.

Second, the estimation of all transition probabilities that are required in our approach provides valuable information about the evolution of the population shares in each class and thus, the mobility in the distribution. Policy makers could use this additional information for planning welfare policies.

Let us assume a system of N economic agents that at each time $t \in \mathbb{N}$, each produces a quantity $y_i(t)$ of income, $i \in \{1, 2, \dots, N\}$.

We classify each agent by allocating it, at each time $t \in \{0, 1, \dots, T\}$, in one of K mutually exclusive classes of income $E = \{C_1, C_2, \dots, C_K\}$ through an allocation map, see Step 2 in Section 3 for a specific choice of the map. This permits the recovering of N time series of states of E :

$$\begin{matrix}
 C^1(0) & C^1(1) & C^1(2) & \dots & C^1(T) \\
 C^2(0) & C^2(1) & C^2(2) & \dots & C^2(T) \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 C^N(0) & C^N(1) & C^N(2) & \dots & C^N(T)
 \end{matrix} \tag{1}$$

where $C^r(t)$ represents the income class of the r -th agent at time t . Individual sample paths are observed in the data (1), which are often called microdata.

Set $(J_t^h = C^h(t))_{h=1,\dots,N; t=0,\dots,T}$ and suppose that such time series are realizations of a discrete time Markov chain with a transition probability matrix \mathbf{P} whose element p_{ij} denotes the probability that an agent now allocated in C_i will enter the next allocation C_j .

For each $C_j \in E$ set

$$y_{C_j} = \frac{\sum_{h=1}^N \sum_{t=0}^T \mathbf{1}_{\{J_t^h=C_j\}} y_h(t)}{\sum_{h=1}^N \sum_{t=0}^T \mathbf{1}_{\{J_t^h=C_j\}}} .$$

It represents the average wealth of agents in class C_j . We assume that each time an agent is in state C_j , it produces a wealth equal to y_{C_j} .

Let us introduce a population structure at $t = 0$,

$$\underline{n}(0) = \{n_{C_1}(0), n_{C_2}(0), \dots, n_{C_k}(0)\}$$

and let $a_{C_j}(\underline{n}(0))$ be the initial share of income due to class C_j :

$$a_{C_j}(\underline{n}(0)) = \frac{n_{C_j}(0)y_{C_j}}{\sum_{h=1}^K n_{C_h}(0)y_{C_h}} = \frac{n_{C_j}(0)y_{C_j}}{\langle \underline{n}(0), \underline{y} \rangle} .$$

We define the process

$$\underline{a}(\underline{n}(t)) = (a_{C_1}(\underline{n}(t)), a_{C_2}(\underline{n}(t)), \dots, a_{C_K}(\underline{n}(t)))$$

which describes the time evolution of the shares of income among the classes of population:

$$a_{C_j}(\underline{n}(t)) = \frac{n_{C_j}(t)y_{C_j}}{\langle \underline{n}(t), \underline{y} \rangle}$$

where

$$\underline{n}(t) = (n_{C_1}(t), n_{C_2}(t), \dots, n_{C_K}(t))$$

and

$$\underline{y}(t) = (y_{C_1}, y_{C_2}, \dots, y_{C_K}).$$

In its static form, Theil's entropy was defined in Theil (1967) by

$$T_e = \sum_{i=1}^K a_i (\log K a_i).$$

Given the population configuration \underline{n} at time $t = 0$ and the vector of average incomes $\underline{y}(t)$, the Dynamic Theil's Entropy is the stochastic process:

$$T_e(t) := \sum_{i=1}^K a_{C_i}(\underline{n}(t)) (\log K a_{C_i}(\underline{n}(t))).$$

The first moment can be evaluated by:

$$\begin{aligned} E[T_e(t)] &= \sum_{i=1}^K \sum_{\underline{n} \in pc} P[\underline{n}(t) = \underline{n}' | \underline{n}(0) = \underline{n}] a_{C_i}(\underline{n}'(t)) (\log K a_{C_i}(\underline{n}'(t))) \\ &= \sum_{i=1}^K \sum_{\underline{n} \in pc} \frac{N!}{\prod_{h=1}^K n_{C_h}'!} \prod_{h=1}^K (P_h(t))^{n_{C_h}'} \left(\frac{\log K a_{C_i}(\underline{n}'(t))}{a_{C_i}^{-1}(\underline{n}'(t))} \right), \end{aligned}$$

where p.c. is the set of all possible population configurations,

$$P_i(t) = \sum_{h=1}^K \frac{n_h(0)}{N} p_{hi}^{(t)}$$

and $p_{hi}^{(t)}$ are the t-step transition probabilities of the Markov chain.

Notice that the static form of the Theil's entropy makes provision for share of incomes that are numbers, whereas that of the dynamic Theil's entropy uses stochastic processes. Additional details on the Dynamic Theil's Entropy can be found in D'Amico and Di Biase (2010).

3. Data and Methodology of Application

To apply the model, we should retrieve the series of microdata concerning agent income evolution. If the individual sample paths (1) are observed, then, the transition probability matrix \mathbf{P} can be estimated by its maximum likelihood estimator; see for example, Theodore W. Anderson and Leo A. Goodman (1957). From the theoretical point of view, the use of microdata poses no problem, but from a practical point of view, the analyst should consider that it is computationally and economically not convenient to work with all sample paths of agents in a country because they are of the order of tens of millions and therefore, sampling techniques should be applied.

For these reasons, we propose here a methodology based only on some macrodata, from which we recover the estimation of the probability matrix \mathbf{P} and the expected value of the Dynamic Theil's Entropy $E[T_e(t)]$. The procedure is simple and can be replicated because input data are widely available. It is important to point out that our procedure, based on macrodata, could be combined with microdata to improve the accuracy of estimation when using the microdata. This is confirmed by previous studies in combining macro- and micro-data in microeconomic models as stated by Guido Imbens and Tony Lancaster (1994). They affirmed "...the reason (of the improvement) is that the aggregate variables give information with little or no sampling error about the averages of the micro variables."

We used macrodata downloaded from the Eurostat website (<http://epp.eurostat.ec.europa.eu/>) on the averages and medians of the net incomes, (see Table 1).

Table 1 Population and Net Income Evolution

Year	Country	Population	Average	Median
2005	Germany	82'500'849	18'214.3	16'393.0
	Greece	11'082'751	11'149.5	9'417.4
	Italy	58'462'375	16'671.0	14'352.0
2006	Germany	82'437'995	17'282.7	15'662.7
	Greece	11'125'179	11'666.3	9'850.0
	Italy	58'751'711	16'647.6	14'523.5
2007	Germany	82'314'906	20'270.4	17'776.5
	Greece	11'171'740	12'130.3	10'200.0
	Italy	59'131'287	17'239.0	15'011.3
2008	Germany	82'217'837	21'086.0	18'309.0
	Greece	11'213'785	12'766.0	10'800.0
	Italy	59'619'290	17'734.0	15'639.0

Source: Authors' representation with data from the EUROSTAT website <http://epp.eurostat.ec.europa.eu/>

We adapt the procedure proposed in D'Amico, Di Biase, and Manca (2012) to gross income distributions to implement the model by means of our macrodata.

Step 1: Recovering the net income distribution

Assume that net incomes follow a lognormal distribution function. This hypothesis is supported by the offices for national statistics of the countries. This hypothesis allows recovering the parameters of the distribution once we know its average and median. If the random variable X is lognormally distributed with parameters μ and σ , then its probability density function is given by

$$f(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\log(x)-\mu)^2}{2\sigma^2}}; \quad 0 \leq x < +\infty, \quad -\infty < \mu < +\infty, \quad \sigma > 0$$

and the corresponding probability distribution function is

$$F(t; \mu, \sigma) = \int_0^t f(x; \mu, \sigma) dx; \quad t \in [0, +\infty).$$

The average is $E[X] = e^{\mu + \frac{1}{2}\sigma^2}$ and the median is $Med[X] = e^\mu$. For example, by using data in Table 1 for Germany in 2005 and solving the system of equations:

$$\begin{cases} e^\mu & = 16'393.0 \\ e^{\mu + \frac{1}{2}\sigma^2} & = 18'214.3 \end{cases}$$

we recover the parameters $\mu = 9.7046$ and $\sigma = 0.459$ and, consequently, the distribution of the income within the country.

Step 2: Building the states

We consider a five states-Markov chain model and allocate each agent in one of the states according to the following rules:

If an economic agent has less than a quarter of the country's average per capita income ($0.25r$), then it is considered poor (class C_1); If the agent has an income between a quarter and one-half of the country's average per capita income, then it is considered medium-poor (class C_2); If the agent has an income between one-half and the country's average per capita income, then it is considered medium (class C_3); If the agent has an income between the country's average per capita income and its double, then it is considered medium-rich (class C_4); If the agent has an income more than the double of the country's average per capita income ($2r$), then it is considered rich (class C_5). The bounds are those proposed by Quah (1996) and represent a reasonable subdivision.

Step 3: Computing net income of classes

For each class C_k , $k \in \{1, 2, 3, 4, 5\}$, we compute the average net income of its agents as the conditional expectation of the income distribution recovered in *Step 1* given that the income is in class C_k :

$$y_{C_1} = \frac{\int_0^{0.25r} x f(x; \mu, \sigma) dx}{F(0.25r; \mu, \sigma)}; \quad y_{C_2} = \frac{\int_{0.25r}^{0.5r} x f(x; \mu, \sigma) dx}{F(0.5r; \mu, \sigma) - F(0.25r; \mu, \sigma)}$$

$$y_{C_3} = \frac{\int_{0.5r}^r x f(x; \mu, \sigma) dx}{F(r; \mu, \sigma) - F(0.5r; \mu, \sigma)}; \quad y_{C_4} = \frac{\int_r^{2r} x f(x; \mu, \sigma) dx}{F(2r; \mu, \sigma) - F(r; \mu, \sigma)}$$

$$y_{C_5} = \frac{\int_{2r}^{\infty} x f(x; \mu, \sigma) dx}{1 - F(2r; \mu, \sigma)}$$

Step 4: Iteration of steps 2 and 3 for the years 2006, 2007, and 2008

The iteration of *Steps 2 and 3* for the years 2006, 2007, and 2008 produces the aggregate data of the number of individuals in each state at each year for each country.

Step 5: Estimation of probability matrix P

Let $N_{C_i}(t)$ be the number of agents in class C_i at time t and $r_{C_i}(t) = \frac{N_{C_i}(t)}{N}$ the proportion of agents in the class C_i at time t .

A method of estimating p_{ij} , while having only these data, is by minimizing the χ squared type expression

$$\sum_{t=2005}^{2008} \sum_{j=1}^5 \left[\frac{\left\{ N r_{C_j}(t) - N \sum_{i=1}^5 r_{C_i}(t-1) p_{ij} \right\}^2}{N \sum_{i=1}^5 r_{C_i}(t-1) p_{ij}} \right]$$

with respect to p_{ij} subject to the restriction $0 \leq p_{ij} \leq 1$ and $\sum_{j=1}^5 p_{ij} = 1$ (see Basawa and Prakasa Rao 1980).

Step 6: Recovering the gross income distribution

For each country, we retrieved the tax rates of the year 2011 and the tax bases. We report them in Table 2. This table summarizes the fiscal systems in force in Germany, Greece, and Italy in 2011. Indeed, the table shows the percentage of taxes computed over the gross income brackets reported on the same line that people in each country have to pay.

Table 2 Individual Income Tax Rates

Germany		Greece		Italy	
Tax %	Tax Base	Tax %	Tax Base	Tax %	Tax Base
0	Up to 8'004	0	1-12'000	23	0-15'000
14	8'005-52'881	18	12'001-16'000	27	15'001-28'000
42	52'882-250'730	24	16'001-22'000	38	28'001-55'000
45	≥ 250'730	26	22'001-26'000	41	55'001-75'000
		32	26'001-32'000	43	≥ 75'001
		36	32'001-40'000		
		38	40'001-60'000		
		40	60'001-10'0000		
		45	≥ 100'001		

Source: Authors' representation with data from <http://www.worldwide-tax.com>.

We considered the following computations to reconstruct the gross income distribution.

Denote by b_i the maximum tax base of the i -th range of income and t_i the tax rate of the i -th range of income. Compute the maximum net income for the i -th range of income as $M_i = (b_i - b_{i-1})(1 - t_i)$ and the maximum cumulated income for the i -th range of income as $\bar{M}_i = \sum_{j=1}^i M_j$. Finally, given a net income d of the year 2005 such that $\bar{M}_i \leq d < \bar{M}_{i+1}$, the corresponding gross income is given by

$$g = b_i + \frac{d - \bar{M}_i}{1 - t_{i+1}}.$$

Step 7: Iteration of Step 6 for the years 2006, 2007, and 2008

Step 8: Repetition of Steps 2 to 5 by considering the gross income distributions obtained at Steps 6 and 7

As explained in the previous steps, the methodology of application assumes that a time homogeneous Markov chain with the transition probability matrix \mathbf{P} describes the income dynamic. Under this assumption the probability

$$p_{ij} = P(\text{agent is in } C_j \text{ at time } t \mid \text{agent was in } C_i \text{ at time } t-1)$$

is independent of t . This implies that, \mathbf{P} is the probabilistic representation of the economy for any time t once the matrix \mathbf{P} was estimated with data from 2005 to 2008. Then, to execute forecasts starting from 2012, we need to fix the fiscal system in force in 2011 and assume that it will not change.

4. Results

We implemented the procedure described in Section 3 for each country.

4.1 Germany

The one-step transition probability matrices for the net and the gross income dynamics were obtained by implementing the systematic procedure proposed in Section 3 and they are the following:

$$\mathbf{P}_1 = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0.896 & 0.068 & 0.036 & 0.000 & 0.000 \\ 0.033 & 0.874 & 0.093 & 0.000 & 0.000 \\ 0.000 & 0.058 & 0.891 & 0.051 & 0.000 \\ 0.000 & 0.000 & 0.082 & 0.895 & 0.023 \\ 0.000 & 0.000 & 0.000 & 0.065 & 0.935 \end{pmatrix} \end{matrix}$$

$$\mathbf{P}_2 = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0.862 & 0.089 & 0.049 & 0.000 & 0.000 \\ 0.020 & 0.928 & 0.048 & 0.000 & 0.000 \\ 0.000 & 0.030 & 0.920 & 0.050 & 0.000 \\ 0.000 & 0.000 & 0.100 & 0.882 & 0.018 \\ 0.000 & 0.000 & 0.000 & 0.078 & 0.922 \end{pmatrix} \end{matrix}$$

Matrix \mathbf{P}_1 expresses the transition probabilities related to the net income dynamic, whereas matrix \mathbf{P}_2 refers to the gross income dynamic. For example, the value 0.896 in \mathbf{P}_1 expresses the probability that an agent in the class of net income C_1 in this year will get to the class of net income C_1 in the next year and the value 0.078 in \mathbf{P}_2 expresses the probability that an agent being in this year in the class of gross income C_5 will get to the class of gross income C_4 in the next year.

As can be seen from the matrices \mathbf{P}_1 and \mathbf{P}_2 , the income dynamic processes show high persistence, indeed, the matrices are both diagonally dominant and monotonic.

Figure 1 shows the forecasting for the Dynamic Theil's Entropy for the net and gross incomes for the next 10 years in Germany. Both forecasts express a decreasing inequality. This means that the economy of Germany is moving towards a fairer wealth allocation among the classes. The introduction of the fiscal system decelerates this convergence towards a fairer income distribution. In particular the comparison between the values computed on the net income (grey line) and those computed on the gross income (black line) allows us to understand how the fiscal system affects the wealth distribution in Germany.

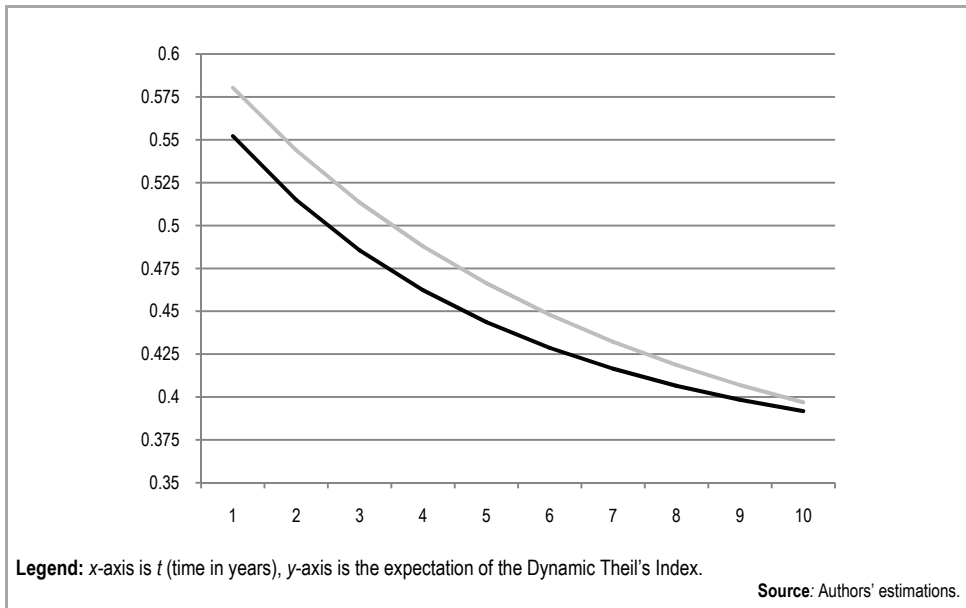


Figure 1 Dynamic Theil's Index for Germany

4.2 Greece

As for Greece, the transition probability matrices are:

$$\mathbf{P}_3 = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0.99450 & 0.00521 & 0.00029 & 0.00000 & 0.00000 \\ 0.00001 & 0.99864 & 0.00135 & 0.00000 & 0.00000 \\ 0.00000 & 0.00003 & 0.99806 & 0.00190 & 0.00000 \\ 0.00000 & 0.00000 & 0.00251 & 0.99746 & 0.00003 \\ 0.00000 & 0.00000 & 0.00019 & 0.00016 & 0.99965 \end{pmatrix} \end{matrix}$$

$$\mathbf{P}_4 = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0.99800 & 0.00120 & 0.00080 & 0.00000 & 0.00000 \\ 0.00100 & 0.99800 & 0.00100 & 0.00000 & 0.00000 \\ 0.00000 & 0.00440 & 0.98700 & 0.00490 & 0.00000 \\ 0.00000 & 0.00000 & 0.00300 & 0.99400 & 0.00300 \\ 0.00000 & 0.00000 & 0.00020 & 0.00040 & 0.99940 \end{pmatrix} \end{matrix}$$

The values show a very strong immobility in Greek society. Matrices \mathbf{P}_3 and \mathbf{P}_4 show that each class represents a cluster. Indeed the probabilities of migration among the classes are very low. This implies that poor agents tend to remain poor

and rich agents tend to remain rich. Both matrices P_3 and P_4 are both diagonally dominant and monotonic. It is noteworthy that the introduction of the fiscal system tends to increase the probability to move towards a poorer class in the next period in spite of a very small quantity of probability.

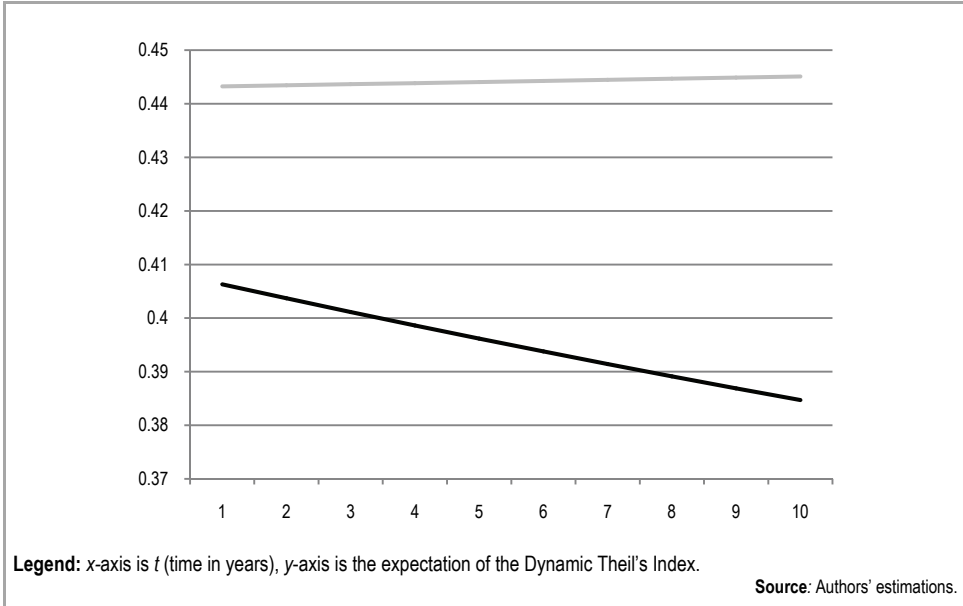


Figure 2 Dynamic Theil's Index for Greece

Figure 2 shows the forecasting of the expectation of the Dynamic Theil's Entropy for the net and gross incomes in Greece. The inequality in the net income (grey line) distribution is approximately constant in time with a tendency to rise slowly. The inequality in the gross income (black line) distribution is markedly lower and exhibits a decreasing path.

4.3 Italy

The transition probability matrices for Italy are the following:

$$\mathbf{P}_5 = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0.762 & 0.203 & 0.035 & 0.000 & 0.000 \\ 0.011 & 0.863 & 0.126 & 0.000 & 0.000 \\ 0.000 & 0.018 & 0.930 & 0.052 & 0.000 \\ 0.000 & 0.000 & 0.065 & 0.905 & 0.030 \\ 0.000 & 0.000 & 0.000 & 0.245 & 0.755 \end{pmatrix} \end{matrix}$$

$$\mathbf{P}_6 = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{matrix} & \begin{pmatrix} 0.701 & 0.264 & 0.035 & 0.000 & 0.000 \\ 0.011 & 0.843 & 0.146 & 0.000 & 0.000 \\ 0.000 & 0.028 & 0.920 & 0.052 & 0.000 \\ 0.000 & 0.000 & 0.072 & 0.896 & 0.032 \\ 0.000 & 0.000 & 0.000 & 0.186 & 0.814 \end{pmatrix} \end{matrix}$$

The matrices are diagonally dominant and monotonic as well. It is noteworthy that, in the net income case, approximately 25% of the agents in class C_5 tend to move towards the poorer class C_4 . This percentage decreases to 18.6% in the gross income dynamic. The opposite happens for the other classes $C_1 - C_4$. In these classes the gross income is less persistent than the net income.

The graphical results shown in Figure 3 lead to the following remarks. The inequality is growing in time for both the net income (grey line) and gross income (black line). Among the considered countries, Italy registers the fastest growth in terms of inequality. The introduction of the fiscal system does not mitigate this increase and it is unable to invert this trend.

A decision maker involved in the policy of economic planning aiming to reduce the initial inequality value of 0.52 has to consider that, *ceteris paribus*, the inequality will grow further than 0.52 and, therefore, an intervention that is more effective should be planned.

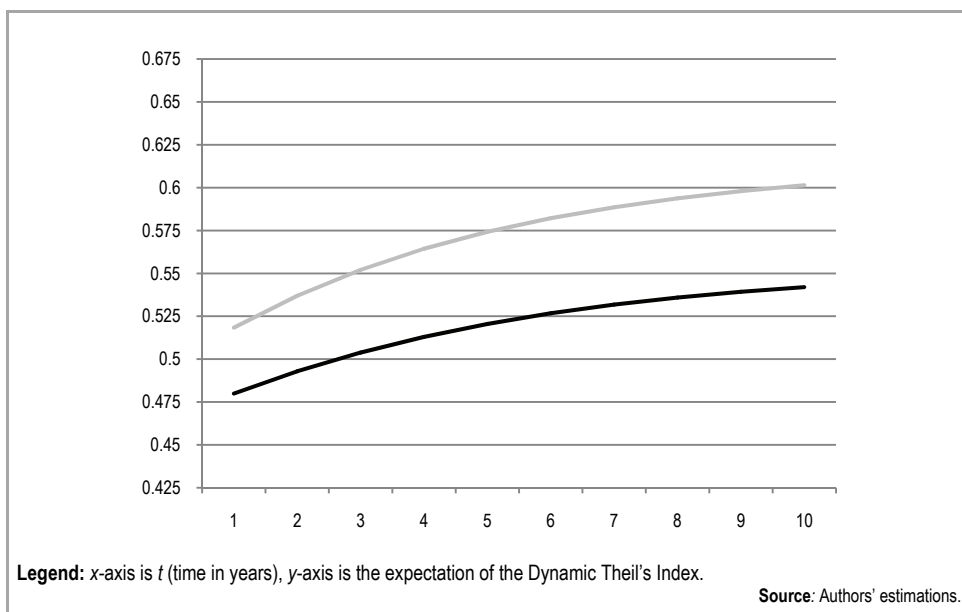


Figure 3 Dynamic Theil's Index for Italy

The inequality in the net income was higher than the inequality in the gross income for all the considered countries and this is a surprising result. The reason is that, in our model, the Theil's index related to the net and gross distributions are computed by changing the size of the population classes. This happens because, for example in the poorest class C_1 (net), we allocated all the agents having less than one-quarter of the net country's average per capita income. Instead when working with the gross distribution, in the class C_1 (gross), we allocated all the agents having less than one-quarter of the gross country's average per capita income. The number of economic agents that belong to class C_1 (net) and class C_1 (gross) is not the same depending on the specific tax rates and tax bases of the countries. This implies a different population configuration for all the classes and for all the years. Obviously, this fact influences the transition probability matrices and the dynamic Theil's entropy. On the contrary, if we had built the model by considering one of the following two alternative schemes:

- a. fix the monetary bounds of the classes ($0.25r$, $0.5r$, r , and $2r$) to be the same in both the case of net and gross incomes; and
- b. fix the monetary bounds such that the numbers of agents in C_i (net) and C_i (gross) are equal;

then, certainly, the inequality in the net distribution would be less than the inequality in the gross distribution.

5. Conclusions

In this paper, we have explained a stochastic model of income inequality. The application of the model requires microdata series of income. We have proposed a general procedure based only on the averages and the medians of the income of each country. These data are widely available and can be considered very accurate. In particular, we analyzed the Eurostat data concerning the three European countries: Germany, Greece and Italy.

We retrieved the tax rates and tax bases for these countries and then, we recovered the gross income distributions. We applied the model to compare the inequalities in the net and gross income distributions. We could measure the wealth redistributive effects caused by the fiscal system from the distances between the curves in the figures.

The results of the application showed different types of temporal evolution of the index. In Germany, the index, forecasting on the gross and net distributions, reveals a decreasing and convex shape. In Greece, the inequality forecast of the net distribution is quite flat, whereas that which is computed on the gross distribution is decreasing. The two curves tend to move away from each other. On the contrary, in Italy, we expect the inequality to grow in time for the net and gross distributions.

Moreover, since the model provides a forecast of the inequality in the population on a given horizon of time, a policy maker can use it to implement a fiscal system that is able to achieve the desired inequality. A study toward this objective will be our next work.

Finally, our model gives a complete description of the probabilistic evolution of shares of income and it is able to study all kinds of functions of the variables $\underline{a}(\underline{n}(t))$. For this reason, the model can be easily extended to the Gini index as it also depends on the shares of income through a specific functional relation that is different from that of the Dynamic Theil's Entropy.

Possible avenues for future development of our model include: a) the application of the model to microdata and the integration of the micro- and macro-data; b) the consideration of an interdependence relationship in population dynamics; c) the construction of a geographical model; and d) the stochastic orders of the processes involved.

References

- Anderson, Theodore W., and Leo A. Goodman.** 1957. "Statistical Inference about Markov Chains." *Annals of Mathematical Statistics*, 28(1): 89-110.
- Arestis, Philip.** 2011. "Fiscal Policy is Still an Effective Instrument of Macroeconomic Policy." *Panoeconomicus*, 2: 143-156.
- Atkinson, Anthony B.** 1970. "On the Measurement of Inequality." *Journal of Economic Theory*, 2: 244-263.
- Athanasopoulos, George, and Farshid Vahid.** 2003. "Statistical Inference on Changes in Income Inequality in Australia." *The Economic Record*, 79(247): 412-424.
- Bajo, Oscar, and Rafael Salas.** 2002. "Inequality Foundations of Concentration Measures: An Application to the Hannah-Kay Indices." *Spanish Economic Review*, 4: 311-316.
- Basawa, Ishwar V., and BLS Prakasa Rao.** 1980. *Statistical Inference for Stochastic Processes*. London: Academic Press.
- Bickenbach, Frank, and Eckhardt Bode.** 2003. "Evaluating the Markov Property in Studies of Economic Convergence." *International Regional Science Review*, 26(3): 363-392.
- Bourguignon, François J.** 1979. "Decomposable Income Inequality Measures." *Econometrica*, 47: 901-920.
- Cowell, Frank.** 1980. "On the Structure of Additive Inequality Measures." *Review of Economic Studies*, 47: 521-531.
- D'Amico, Guglielmo, and Giuseppe Di Biase.** 2010. "Generalized Concentration/Inequality Indices of Economic Systems Evolving in Time." *Wseas Transactions on Mathematics*, 9(2): 140-149.
- D'Amico, Guglielmo, Giuseppe Di Biase, and Raimondo Manca.** 2012. "Income Inequality Dynamic Measurement of Markov Models: Application to Some European Countries." *Economic Modelling*, 29: 1598-1602.
- De La Vega, Casilda L., and Ana M. Urrutia.** 2005. "Path Independent Multiplicatively Decomposable Inequality Measures." *Investigaciones Economicas*, 29(2): 379-387.
- Imbens, Guido W., and Tony Lancaster.** 1994. "Combining Micro and Macro Data in Microeconomic Models." *The Review of Economic Studies*, 61(4): 655-680.
- Kooleman, Xander, and Eddy Van Doorslaer.** 2004. "On the Interpretation of a Concentration Index of Inequality." *Health Economics*, 13(7): 649-656.
- Prokopijević, Miroslav.** 2010. "Euro Crisis." *Panoeconomicus*, 3: 369-384.
- Quah, Danny T.** 1993. "Galton's Fallacy and Tests of the Convergence Hypothesis." *The Scandinavian Journal of Economics*, 95(4): 427-443.
- Quah, Danny T.** 1994. "One Business Cycle and One Trend from Many, Many Disaggregates." *European Economic Review*, 38: 605-613.
- Quah, Danny T.** 1996. "Empirics for Economic Growth and Convergence." *European Economic Review*, 40: 951-958.
- Reza, Ranjpour, and Karini T. Zahra.** 2008. "Evaluation of the Income Convergence Hypothesis in Ten New Members of the European Union. A Panel Unit Root Approach." *Panoeconomicus*, 2: 157-166.
- Shahrestani, Hamid, and Bijan Bidabad.** 2010. "An Implied Income Inequality Index Using L_1 Norm Estimation of Lorentz Curve." *Global Journal of Business Research*, 4(1): 29-46.
- Shorrocks, Anthony F.** 1984. "Inequality Decomposition by Population Subgroups." *Econometrica*, 52: 1369-1385.

- Stojanović, Božo.** 2011. "Uncertain Future of the European Union." *Panoeconomicus*, 4: 559-570.
- Theil, Henri.** 1967. *Economics and Information Theory*. Amsterdam: North Holland.

THIS PAGE INTENTIONALLY LEFT BLANK